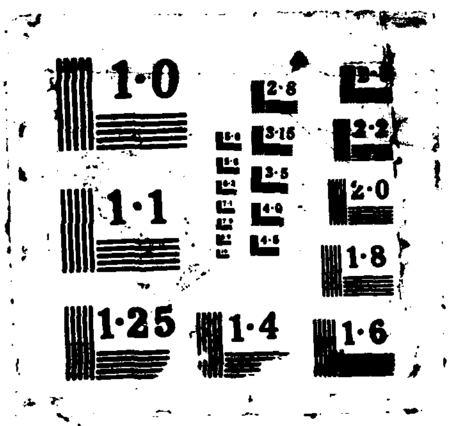
AD-A185 269 THE USE OF MATHEMATICAL PROGRAMMING AND RESPOOSE SURFACE METHODOLOGY IN O.:(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI.. R F HAILE DDC 86 AFIT/GOR/OS/86D-4

1/2

UUCLASSIFIED F/G 15/5 NL


1·0
2·8
2·2
2·0
1·1
1·8
1·25
1·4
1·6

AD-A185 269

OTIC FILE COPY

AFIT/GOR/OS/86D-4

THE USE OF MATHEMATICAL PROGRAMMING AND RESPONSE SURFACE METHODOLOGY IN OPTIMIZING THE AIRLIFT FORCE STRUCTURE IN A FAR EASTERN THEATER OF OPERATION

THESIS

Raymond F. Haile
Major, USAF

AFIT/GOR/OS/86D-4

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

AD-A185 369

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; Distribution Unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) AFIT/GOR/OS/86D-4	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION School of Engineering Air Force Inst. of Tech. 6b. OFFICE SYMBOL (If applicable) AFIT/ENS	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State and ZIP Code) Wright-Patterson AFB, OH 45433	7b. ADDRESS (City, State and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State and ZIP Code)	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT NO.
11. TITLE (Include Security Classification) See Box 19	
12. PERSONAL AUTHOR(S) Raymond F. Haile, Maj, USAF	

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
MS Thesis	FROM ______ TO ______	1986 December	155

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	Response Surface Methodology, Experimental Design
12	02		Deterministic Models, Airlift, C-17

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Block 11:

Title - THE USE OF MATHEMATICAL PROGRAMMING AND RESPONSE SURFACE METHODOLOGY IN OPTIMIZING THE AIRLIFT FORCE STRUCTURE IN A FAR EASTERN THEATER OF OPERATION

Advisors - Joseph R. Litko, Maj, USAF
- Palmer W. Smith, Dr.

Approved for public release: IAW AFR 190-1.
LYNN E. WOLAVER 3 Sep 87
Dean for Research and Professional Development
Air Force Institute of Technology ~~(ATC)~~
Wright-Patterson AFB OH 45433

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Joseph R. Litko, Maj, USAF	22b. TELEPHONE NUMBER (Include Area Code) (513) 255-3362 22c. OFFICE SYMBOL AFIT/ENS

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

UNC ASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

This thesis refines an earlier demonstrated methodology which combines response surface methodology and experimental design concepts to describe the output of a deterministic model. In an effort to maximize the combat power delivered to a theater commander, several critical factors are varied to determine their impact upon the delivery of men and equipment to the objective area. This model does not use metric ton-miles per day as a measure of effectiveness, but rather it places a time dependent value on the delivery of combat units to the objective area. The deterministic model includes 168 equations and 288 different variables. A response surface was generated using an appropriate experimental design, with the response surface predictions averaging greater than a 96 percent level of accuracy. The analysis section demonstrates the advantages and disadvantages of the sensitivity analysis normally associated with linear programming, and the advantages and limitations of using response surface methodology to generate a multidimensional analysis of the model. While the optimum airlift force mix may be heavily scenario dependent, this research provides additional insight into the complexities of developing an appropriate airlift force to meet future Department of Defense requirements.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

AFIT/GOR/OS/86D-8

THE USE OF MATHEMATICAL PROGRAMMING AND RESPONSE SURFACE METHODOLOGY IN OPTIMIZING THE AIRLIFT FORCE STRUCTURE IN A FAR EASTERN THEATER OF OPERATION

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements of the Degree of

Master of Science in Operations Research

Raymond F. Haile, M.A., B.S.

Major, USAF

December 1986

Acknowledgments

I want to extend a special word of thanks to both of my advisors, Major Joseph Litko and Dr. Palmer Smith. Their knowledge of the subject matter, unending patience, and insightful suggestions were much appreciated.

I also especially want to thank my dear wife, Nancy and our two wonderful children, Cortney and Michael, for their continuing patience and understanding. Their support and words of encouragement were a tremendous help to me and reemphasized the importance of a close and loving family.

Raymond F. Haile

TABLE OF CONTENTS

List of Figures

List of Tables

List of Terms

ACL	-	Allowable cabin load
ACRA	-	Airlift Concepts and Requirements Agency
Airdrop	-	The delivery of personnel, supplies, or equipment by means of parachute
Airland	-	The delivery of personnel, supplies, or equipment by means of the aircraft landing and manual downloading of the cargo
ALCE	-	Airlift Control Element
APOD	-	Aerial Port of Debarkation
APOE	-	Aerial Port of Embarkation
CONUS	-	Continental United States
CRAF	-	Civil Reserve Air Fleet
FOL	-	Forward Operating Location
GAO	-	Government Accounting Office
HTTB	-	High Technology Test Bed
MAC	-	Military Airlift Command
MAW	-	Military Airlift Wing
MHE	-	Materials Handling Equipment
MOB	-	Main Operating Base
MOE	-	Measure of Effectiveness
PAA	-	Primary Aircraft Authorized
RSM	-	Response Surface Methodology
SAC	-	Strategic Air Command

Abstract

This thesis refines an earlier demonstrated methodology which combines response surface methodology and experimental design concepts to describe the output of a deterministic model. In an effort to maximize the combat power delivered to a theater commander, several critical factors are varied to determine their impact upon the delivery of men and equipment to the objective area. The effects of time, materials handling equipment ~~(MHE)~~ capability, airfield ramp space, distance between the Aerial Port of Debarkation ~~(APOD)~~ and Forward Operating Location ~~(FOL)~~, the availability and capabilities of different aircraft, and the impact of aircraft attrition are examined in the model. This model does not use metric ton-miles per day as a measure of effectiveness, but rather it places a time dependent value on the delivery of combat units to the objective area. The deterministic model includes 168 equations and 288 different variables. A response surface was generated using an appropriate experimental design, with the response surface predictions averaging greater than a 96 percent level of accuracy. The analysis section demonstrates the advantages and disadvantages of the sensitivity analysis normally associated with linear programming, and the advantages and limitations of using response surface methodology to

generate a multidimensional analysis of the model. While the optimum airlift force mix may be heavily scenario dependent, this research provides additional insight into the complexities of developing an appropriate airlift force to meet future Department of Defense requirements.

THE USE OF MATHEMATICAL PROGRAMMING AND RESPONSE SURFACE METHODOLOGY IN OPTIMIZING THE AIRLIFT FORCE STRUCTURE FOR A FAR EASTERN THEATER OF OPERATION

I. Introduction

Background

In 1975, the commander of the Military Airlift Command stated the enhancement of our airlift capability is a critical step in acquiring an efficient airlift force. (5:2-10) In order to protect its vital security interests, the United States must be able to support its forces overseas. In the early stages of a major conflict, airlift will play a dominant and vital role in delivering the men and equipment required for the operation. An effective airlift force must be able to meet strict time constraints and it should be able to deliver combat units directly from their place of origin to the employment area. The current airlift force structure does not possess that capability (4:iii).

This type of concern precipitated the Congressionally Mandated Mobility Study (CMMS), which documented the

requirement for additional airlift capability, especially of outsized cargo. As a short term solution, in January of 1982 the Department of Defense announced the purchase of 50 improved C-5B aircraft. When the last C-5B is delivered in 1989, long range intertheater airlift capability will reach 46 million ton-miles per day, still 26 per cent short of the 66 million ton-miles per day goal established by the CMMS (24:28-29).

As an additional factor, the remaining airlift fleet is in need of modernization. The C-141, the current workhorse of intertheater airlift, is over 20 years old and will either need to be replaced around the turn of the century or be upgraded with a life extension program. The primary intratheater aircraft, the C-130, has several models which are rapidly approaching the end of their service life (21:14). Airlift planners must now determine how to effectively meet the need for a replacement aircraft capable of fulfilling the vital intratheater mission.

In 1980, the report of the Secretary of the Air Force and Air Force Chief of Staff to Congress stated that "the current airlift deficiency is judged to be the greatest problem the Air Force faces in executing the national military strategy". (34:17)

While the CMMS was still in progress, airlift planners at the Air Staff and the Military Airlift Command formed a C-X Task Force to develop the operational concept and to determine what capabilities would be desirable in a future

aircraft. This Task Force produced the Preliminary System Operational Concept (PSOC) which described the intended purpose of the C-X. The PSOC developed several scenarios and focused upon the mission required of the aircraft for each scenario. The mission requirements were determined and passed to the aircraft manufacturers, who were given the flexibility to design the aircraft to meet the needs of the Air Force. Boeing, Lockheed, and McDonnell Douglas all responded to the request for proposal, and in 1981, the Source Selection Evaluation Board convened to examine all the proposals. In August of the same year the Secretary of the Air Force selected the McDonnell Douglas design as the winner. The design has subsequently been designated the C-17 (24:28).

The C-17's ability to deliver cargo from onload bases in the United States directly to the operational forces in the field adds a new dimension to airlift responsiveness and highlights why acquisition of the C-17 has become a major priority for MAC (1:37). The flexibility of the C-17 in meeting the intertheater, intratheater, and outsize cargo carrying requirement has made the C-17 the cornerstone of MAC's effort to meet future airlift requirements. MAC must now convince Congress that the C-17 is a better choice than the C-5B, even though 50 C-5B's have already been approved by Congress.

The major disadvantage to the C-17 is its acquisition cost. At over $100 million per aircraft, the added

capability does not come cheap. The C-17 can deliver outsized cargo directly to the objective area, but battlefield commanders might be reluctant to commit such an expensive resource to a hostile environment (1:51). While the C-5B costs over $140 million per aircraft, it can carry twice as much cargo by volume, 36 pallets versus 18 for the C-17, and 40 per cent more cargo by weight, 242,000 pounds versus 172,000 pounds for the C-17. Yet, in a report from the GAO to the Chairman of the Committee on Armed Services, the GAO stated that the Air Force feels that neither the C-5A or the C-5B can routinely and safely land or takeoff from small austere airfields. Although Lockheed disagrees, the Air Force feels that these conditions would be operating the crew and aircraft near their limits with small margin for error. This report stressed the improved capability of the C-17, such as its smaller size, better maneuverability, better combat offload capability, and airdrop capabilities, and how these improvements are vital to the accomplishment of the entire airlift mission (34:2). Due to its size and design characteristics the C-5B would be restricted to operating within only well developed airfields and would probably not operate in a hostile environment. Outsized cargo would have to travel overland to get to the objective area which means less threat to the aircraft but a longer time delay involved in getting the required equipment to the battlefield commanders. As stated by Colonel Bullard, during his assignment to the Air War College, "unless the men,

equipment, and supplies are delivered to the combat area, only a marginal utility will be achieved by transporting them to the general theater of operation." (4:23) There definitely exists a tradeoff between capability, costs, effectiveness, and vulnerability, and the Air Force and Congress must agree upon which factors have priority.

Research on this topic and methods capable of quantifying the relative advantages of each aircraft have been discussed only peripherally.

After reviewing such factors as military utility, program costs, manpower requirements, force stabilization, and force modernization, the CMMS recommended a force structure relying heavily upon the C-17. The CMMS stated this airlift force would provide the United States with the best ability to rapidly deploy and sustain its fighting forces in pursuit of national objectives (8:V-12).

A paper submitted at the U.S. Army War College assessed the capabilities of the C-17 had it been available for use in two actual airlift operations. The first operation, AHUAS TARA83, was a combined operation in Honduras, and the second, Urgent Fury, was the U.S. rescue mission to Granada. In both cases the C-17 was shown to be more effective and efficient than the aircraft actually used in the operations (13:14-29).

In a paper submitted at the Air Command and Staff College, several advantages and disadvantages of the C-17 were discussed. In spite of reservations about using the C-17 in a moderate threat environment, this paper recommends

purchasing the maximum number of aircraft (approximately 300) (1:56-57).

As further evidence of the utility of the C-17, a thesis submitted by Major John Stone stressed the "direct delivery" concept of the C-17. In his conclusion he stated that the opponents who denounce the C-17 because its payload capacity is less than the C-5 do not understand that cargo capacity is of little value if the cargo is not delivered in a timely and efficient manner to where it is needed (32:91).

As pointed out by Major Soligan, while assigned to the Air Command and Staff College, airlifts' role in the Airland Battle still needs to be researched and defined. He emphasized the importance of the Airlift Concepts and Requirements Agency (ACRA) and its role in meeting with the theater commanders to resolve the proper role of airlift in the deployment and resupply effort required to develop doctrine and support the desired concept of operations (30:53).

None of the articles thus far have dealt with the issue of airlift force structure using an analytical or simulation model. In 1984 Army Captain James Cook, an AFIT/GST student, demonstrated his methodology using Sequential Goal Programming and Response Surface Methodology to determine how the interaction of unit weight, combat attributes, logistics needs, and airlift resources could be jointly optimized for a given contingency. By developing a model which he demonstrated using 212 variables and 136 constraints, he was

able to analytically determine the optimal force mix or incremental advantage in deployed power attainable by an incremental change in airlift resources (7:4).

It is the purpose of this investigation to expand and refine the methodology used by Captain Cook to optimize the airlift force structure required in a Far Eastern theater of operation.

Problem Statement

There is currently no analytical model which accurately and fully quantifies the airlift force mix which would optimize the combat power delivered to a battlefield commander in a Far Eastern theater of operation.

Research Objective

The primary objective of this research effort is to refine an earlier demonstrated methodology that quantifies the value of C-5 and C-17 aircraft in intertheater airlift and the C-130 and C-17 in an intratheater role. The capability of each aircraft to meet mission requirements is a function of several major factors including the quantity and capabilities of the specific aircraft and the survivability of the aircraft in a threat environment. The impact of the different factor levels, including quantity of aircraft available, quantity of materials handling equipment (MHE)

available, distance between APOD and FOL, available ramp space, and aircraft attrition, upon the measure of effectiveness of the model and the relative value of the C-5, C-130, and C-17 needs examination and quantification in order to determine an efficient and effective airlift force mix for a specific scenario.

Scope

The determination of an appropriate airlift force mix is heavily influenced by the scenario requirements. The selection of a Far Eastern theater of operation is intended to represent the type of scenario which the United States must be prepared to meet; a scenario which combines such critical factors as intertheater and intratheater distances, airfield and MHE saturation, different aircraft capabilities, and the presence of a threat environment. Several limitations to the scenario are listed in the scenario development chapter. The linear programming model and the response surface equations are intended to demonstrate the methodology of effectively utilizing RSM and experimental design concepts to explain the output of a deterministic model. While necessarily restrictive, the methodology is intended to provide valuable insight into the complexities of determining airlift force structure requirements.

Format

The format for this thesis consists of the following major areas. Chapter II covers the general methodology by reviewing response surface methodology and experimental design, followed by a discussion of recent applications of these concepts. Chapter III develops the formulation of the 168 constraints and 288 variables required for the linear programming model. The next chapter, Chapter IV, introduces the data base used to develop the scenario. Chapter V is an analysis of the results, including the development of the experimental design and response surface equations. Chapter VI contains a summary of conclusions and a list of possible recommendations for further research.

II. General Methodology

Introduction

The overall objective of this study is to refine the methodology established by Captain Cooke in order to maximize the combat power delivered to a Far Eastern theater of operation as a function of time. While considering such factors as aircraft capabilities, scenario limitations, and attrition of aircraft, this objective can be stated as one overall goal: maximize combat power delivered while minimizing the time required to deliver the army's assets to the objective area.

Response Surface Methodology

Response Surface Methodology (RSM) is a mathematical method which permits a particular set of mathematical and statistical tools to be used by researchers to solve certain types of problems, especially in industrial research. This concept has been used extensively in industry to examine factor relationships and to determine which combinations of factors result in the optimum output (29:58).

Mathematical models formulated by researchers consist of a number of input variables, or independent variables, which describe the output or response of a given experiment. The response surface generated by the mathematical equations provides a picture of the influence

of the various input variables upon the primary measure of interest (response variable).

Ideally, the researcher's model would include all the different variables that affect the output response. Realistically, constructing an appropriate mathematical model to represent a real world problem will most probably be an iterative process where variations to the model will eventually result in the model response approximating the value of the real response. In a complicated process, the potential for not including a critical factor in the development of a deterministic model can be significant. The exclusion of one or more critical factors in a model produces a model with biased results. The factor or factors not considered in the model bias the model results and produce conclusions that differ from actual results. A good mathematical model will tend to minimize bias error as well as the second type of modeling error, variance error.

Variance error is caused by an inability to hold certain factors constant during an experiment. In the chemical industry, the need to minimize variance error is critical to the successful interpretation of a given experiment. The method of least squares regression allows the researcher to minimize error by determining whether error is due to an incomplete list of the factors or to poor measurement methods.

RSM is used here to determine the least number of measurements which are needed to accurately fit the model to the data. If RSM can be used to accurately determine an appropriate mathematical model to represent the output of an experiment, then the researcher is no longer required to run the entire experiment in order to analyze possible outcomes from new combinations of factors. As long as factor combinations are chosen which remain within the bounds of the experiment, the response of the experiment can be closely approximated using the mathematical model.

Box and Wilson first developed the theory of Response Surface Methodology in 1951. Throughout the years several useful experimental designs have been developed to expand the usefulness of RSM for various purposes. The most common and easily understood designs are for first-order and second-order models. A first-order model is linear in form. It is said to be linear because no parameter appears as an exponent or is multiplied or divided by another parameter. The independent variable is also considered linear because it appears only in the first power. Second-order models are slightly more complex and include the use of quadratic variables and variables representing two-way interactions. The following equations illustrate first and second-order models:

Y = value of response variable

B_0 and B_1 = parameters (variables)

x = value of independent variable

ϵ = random error term

$$Y = B_0 + B_1x_1 + \epsilon$$

$$Y = B_0 + B_1x_1 + B_2x_2 + B_{12}x_1x_2 + B_{22}X_2^{\;2} + \epsilon$$

In analyzing the coefficients of a given model, it is desirable to have coefficients which are uncorrelated. Correlation refers to the influence one variable has upon the value of another variable in the equation. In 1960 Box and Behnken suggested possible new designs which possessed a high degree of orthogonality. Orthogonal variables are uncorrelated; therefore, the various terms in the mathematical equation are independent of each other. This allows the various coefficients to be compared to each other independently or as ratios. In the Box and Behnken designs only the constant term B_0 and the quadratic estimates are correlated to one another, but current data has shown that the correlation between these coefficients has been very small and has not presented any major problems in practical applications (29:60).

As stated previously, the error from a model is produced by a combination of bias error and variance error. Variance error reflects a measurement error whereas bias error is caused by an improperly fitting model probably caused by the omission of critical

variables in the mathematical equations describing the experiment.

In the past, most of the discussion on experimental design has centered around the importance of minimizing this variance error. Very little work has been accomplished in the area of minimizing bias error. Myers summarizes the work accomplished by Box and Draper in his book "Response Surface Methodology". (22)

The earlier applications of RSM, which emphasized variance reduction, centered around chemical experiments and the need to minimize the error caused by improper methods of measurements in the laboratory. The chemical industry has made good use of RSM in predicting outcomes and in determining the effects of interaction upon output responses.

As the use of RSM increases, there is a growing need to emphasize the importance of minimizing bias error. Analytical math models which use the same inputs produce results with no real variance. The outputs of these deterministic models do not change if the input values do not change; therefore, variance is not a factor. Bias error now becomes the prevalent type of modeling error, and research continues into methods of minimizing it. Another important factor to consider when constructing a response surface is selecting an appropriate experimental design.

Experimental Design

When selecting an experimental design to construct a response surface, several guidelines need to be emphasized. As stated by Myers, the factors must be quantitative and continuous, the function must be able to be approximated using a low-order polynomial, and the independent variables $x_1, x_2, \ldots x_k$ should be highly controlled and be measurable with only a negligible amount of error (22:62).

Other requirements for a good experimental design include the ability to accurately estimate the coefficients while not requiring an inordinate number of observations. In order to properly fit a second-order model, the variables in the model must be evaluated at three different levels, usually at a low, high, and intermediate value (20:462). Another important feature is the designs' rotatability. As stated by Myers, "A design is said to be rotatable when the variance of the estimated response, that is, the variance of "y", is a function only of the distance from the center of the design and not on the direction." (22:139) As a result, points in the factor space which are the same distance from the origin, are treated as being equally important. Or stated another way, rotatability insures a symmetric generation of information in the space defined by the variables (3:457).

Another desirable feature of an experimental design is to produce coefficients which are uncorrelated. As stated in the section on response surfaces, uncorrelated coefficients are produced by orthogonal designs. In an orthogonal design the columns of the X matrix are orthogonal to each other so in a completely orthogonal design the (X'X) matrix would produce zeroes for all the off-diagonal elements. The coefficients would therefore be independent and uncorrelated with each other. The three-level rotatable central composite design selected for this thesis was developed by Box and Behnken (3:461) and is nearly orthogonal as evidenced by the correlation matrix listed in Table X. Only the constant term β_0 and the quadratic terms are correlated with each other and as previously stated, the degree of correlation is extremely small and does not affect the significance of the design output (29:60). The coded and non-coded designs are listed as Appendix A and Appendix B respectively. The next section of this chapter discusses several recent applications which have incorporated the concepts of response surface methodology and experimental design.

Military Applications of RSM and Experimental Design

Within the last several years, a number of papers have been written which applied the methods of RSM and Experimental Design to various military problems. The next few pages will discuss the implications and conclusions of several of these efforts.

One of the first important applications which combined RSM, mathematical programming, and experimental design, was written for SAC in 1975 by LTC Smith and Dr. Joseph Mellichamp. They established a methodology which will be employed in this thesis, which allows for multidimensional impact analysis of military problems. The article, which was first published in 1975, pointed out the classical sensitivity analysis of varying one parameter at a time has some major disadvantages. First, only one parameter can be evaluated at a time. Second, the current method does not allow for any numerical ranking of the importance of each parameter to the total solution. Third, no information can be provided about the interrelationships that may exist between the different factors under study.(29:56) RSM will allow a multidimensional analysis of the same problem. Using the correct experimental design to optimally select a number of combinations of the input parameter values, linear or quadratic programming can find the optimum value of the response variables. Linear regression can then be used to determine the coefficients of the mathematical equation defining the response surface (29:58). As long as the factor combinations are within the range of the parameters used to develop the initial equation, the mathematical equation can then be used as a method of providing sensitivity analysis for the true response surface.

The second paper to be discussed is the thesis of Captain James Cooke. His work forms the foundation for this thesis effort. He developed the methodology which will be refined and expanded to determine the optimum combat power delivered to a specific theater of operation, for a specified time period, and within certain sustainability levels. He related the effects of intertheater and intratheater movement and the deployable unit's capabilities to the optimization of combat power delivered to a given objective area. Unit weights, combat power attributes, resources, and in-theater constraints were used to find global optimum solutions (7:4).

Response Surface Methodology was used as a method of sensitivity analysis by examining the effects of several independent variables upon combat power delivered to the front. The flexible response surface was generated using a fractional factorial design based upon the need to maintain orthogonality between variables and to minimize bias error within the model. Captain Cooke demonstrated his methodology by using an RSM model containing five variables, which consisted of the number of C-5A and C-17 aircraft, APOD size, materials handling equipment, and distance from the APOD to the forward operating location. He analyzed the incremental advantage in deployed combat power attainable by an incremental change in airlift resources (7:4). It is the purpose of this thesis to apply Cooke's methodology to a specific scenario. In an

effort to further examine the impact of certain variables within the mathematical model, the additional variables of C-130 availability and aircraft attrition, and the impact of aircraft attrition upon the amount of combat power delivered, will be examined in the response surface model. Previous discussions in airlift force structuring have indicated the importance of the scenario in determining the optimal force mix, and this scenario will center around the deployment of combat units from CONUS to the Republic of Korea.

The different potential applications of RSM can be demonstrated by another thesis presented in December of 1984 by Captain Manacapilli. He combined the use of RSM, economic production functions, economic theory, deterministic models, and lagrangian techniques to help identify cost effective strategic force mixes. His methodology was used to fit economic production functions to the response surface of an unclassified linear programming nuclear exchange model. The thesis presented a methodology which combined the applications of economics with RSM to analyze the output of a deterministic model. His research showed that if economic production functions are properly fitted to a response surface, the information gained can provide useful insight into the production process itself (15:6.4).

Another example of the far reaching impact of combining mathematical programming with response surface methodology was the work done by Captain Graney, who examined the use of RSM as an aid to multi-dimensional decision making. Graney's study presents the optimization methodology for a deterministic, mathematically modeled system as defined by the consecutive levels of one measure of effectiveness (MOE) and the optimization of a second goal or mission. This methodology allows the decision maker the option of a large number of alternatives. It is capable of comparing the MOE's throughout the entire range of alternatives and allows the use of non-commensurate measures of effectiveness (11:79). Graney demonstrates his methodology by examining an aggregated military force structure problem as defined by a linear programming arsenal exchange model. The two measures of effectiveness are counterforce and countervalue, and the system consists of five weapons systems and ten target classes. As noted by the author, the number of feasible alternatives for the system exceeds three billion, but optimal effectiveness of these alternatives can be approximated through the use of response surface methodology.

Using an appropriate fractional factorial design, a response surface was generated for each MOE. The methodology allows for a tradeoff between the two MOE's by optimizing the second measure of effectiveness as it is constrained to the contour lines of the first measure of

effectiveness. This thesis demonstrated the usefulness of RSM in examining the alternatives in a multi-criteria comparison of alternatives and approximating the optimal effectiveness of those alternatives.

Three recent thesis efforts have attempted to develop interactive user-friendly computer models to assist users in applying and analyzing complex deterministic or simulation models.

The first model was developed by Captain Tate in response to Captain Cooke's thesis recommendation that his mathematical model be developed into a user-friendly computer package.

Captain Tate developed an interactive goal programming model designed to analyze the rapid deployment of units of the armed forces to a selected crisis area. The mathematical model used by Tate in creating his computer model, DEPLOY, was taken largely from the mathematical model developed by Captain Cooke, discussed earlier in the text. Tate developed some alternative methods for describing Cooke's mathematical equations, but for the most part, his contributions center around his creation of a user-friendly model which would capture the complex mathematical relationships established by Cooke. The allocation routine, PAGP, needs refinement in order to correctly interface with the computer package DEPLOY, and this enhancement was presented in the thesis as a recommendation for further research (33:7-4).

Nevertheless, Tate did develop Cooke's mathematical model into a computer package which analyzed the accomplishment of Army objectives as well as determined the minimum force structure required to accomplish the goals set by the Army. A potential advantage of this model is its ability to perform two types of analysis. The first determines the degree to which the Army's goals can be met, given that the number and types of available aircraft and deployable units are predetermined parameters. The second type of analysis determines the minimum force structure required to meet the combat power goals set by the Army (33:7-2).

The second computer model, developed by Mr. Meitzler, implemented a user-friendly program module into a relatively new deterministic simulation model, Budget/Readiness Analysis Technique (BRAT). The model was developed by the Studies and Analysis Directorate of the Acquisition Logistics Center (AFALC/XRS), and it converts the logistics resources of maintenance manpower, support equipment, and spares into a sortie rate to determine a system's state of mission readiness. He applied the techniques of response surface methodology, experimental design, and multiple regression to develop a module which uses low order polynomials to approximate the response surface of the BRAT model. The response surface can be used to provide the capability to quickly perform sensitivity analysis on the logistics resources and

simulation parameters (18:vii,24). He discusses the importance of selecting the correct experimental design based upon any prior knowledge of the form of the response surface. First and second order designs were examined, and the importance of the design defining the boundaries of the experimental region was examined. A first order surface would use a 2^k design where each k-factor is taken at two levels, and a quadratic or second order design using a 3^k design would insure each K-factor is taken at three levels. The major contribution of this thesis lies in the implementation of a user-friendly computer module into the BRAT module which provides the user the capability to use RSM to analyze the outputs of the model and to perform sensitivity analysis on the logistics resources and simulation parameters (18:24).

A third interactive computer package was developed by 1Lt Sparrow. She designed a user-friendly response surface methodology computer package which could be attached to any Fortran based simulation model. The model was designed to produce a response function to accurately predict the relationship between the input variables and the response variable. The computer package was developed to be implemented on the VAX 11/780, which can run the simulation models and any required Fortran subroutines. The package was designed to be user-friendly and offered five different design types, but it did not include the Box and Behnken designs which will be used for this thesis

(31:106). Their inclusion into the program was listed as a possible enhancement to the current model.

The last several papers discussed the capabilities of response surface methodology to predict the output of a complex model. In order for RSM to achieve this goal, the response surface must be developed with the appropriate experimental design. The importance of selecting an appropriate design, containing certain important characteristics, was analyzed by Captain Ishihara.

He evaluated the effectiveness and efficiency of several bias and variance minimizing experimental designs. He used several experimental designs to develop response equations for a deterministic nuclear exchange model. The following criteria was used to evaluate the design points: 1) the total number of design points, 2) the number of terms in the response equation, 3) the accuracy of the fit of the response equation, 4) the orthogonality of the design, and 5) the rotatability of the design (14:ix). He also evaluated the various equations' ability to estimate the true surface and to predict future responses from a given set of input data. This paper concluded that a second order polynomial can be an excellent choice for estimating the response surfaces of deterministic nuclear models. The level of accuracy of a modified equation depends upon the number of design points and upon the degree of rotatability of the design (14:4-2). The response equation should contain as few

cross product and squared terms as possible. The experimental design's accuracy should improve as the number of design points increased, as long as the design points were chosen from throughout the entire surface. Additionally, if the region being investigated is spherical, a rotatable design improved the accuracy of the overall response equation (14:4-2). Another conclusion of Ishihara is that specific conditions must be met in order for the coefficients of a modified response equation to accurately represent the marginal effects of change of the dependent variable caused by incremental changes to the independent variables. These conditions include: 1) a high degree of multicollinearity should not exist between the independent variables, 2) the main effect terms of the response equation should dominate the squared and interaction terms, and 3) the modified response surface should be able to accurately represent the true surface (14:4-3). These conclusions reaffirm the importance of incorporating a good experimental design into the development of the response surface.

All of these papers demonstrate the potential analytical value of combining response surface methodology with an appropriate experimental design to develop an accurate and efficient method of examining the effects of critical factors upon the outputs of deterministic models.

III. Model Development

Introduction

This chapter will be used to develop and explain the mathematical expressions describing the goals and constraints used in the model. The formulations of the goals and constraints for this model are taken largely from the thesis of Captain Cooke. The variables and notation for considering the constraints over a period of days, rather than for every day, are taken from the previous work of Captain Tate. While Cooke formatted his constraints for a daily operation, Tate transformed those constraints to be applicable over a given period length. There are minor differences for a variety of constraints and the data base describing military hardware capabilities, and these differences will be fully examined in this chapter. The variables, multipliers, and subscripts will be discussed first, followed by an examination of the development of the goals and constraints. After each constraint is derived, the number of that type constraint required for the model will be given, followed by the formulation as it applies to this particular model. The end of the chapter will include a summary of all the goals and constraints required for this particular model.

Variables

The variables as formulated by Captains Cooke (7:46-76) and Tate (33:chap3,1-31) are defined as follows:

Variables Used:

P(k,l): variables used to signify excess units at the FOL which will be available for use at the start of the next period. "k" refers to the cargo type and "l" refers to the period.

PS(k,l): variables used to signify excess supplies at the FOL which will be available for use at the start of the next period.

A(k,l): variables used to signify excess units at the APOD available for use at the start of the next period.

AS(k,l): variables used to signify excess supplies at the APOD which will be available for use at the beginning of the next period.

Sl: the amount of supplies transshipped in period l from the APOD to the FOL. [tons/period]

x(i,j,k,l): the number of delivered aircraft loads. [ac/period]

u(y,m,l): the number of units deployed.[units/period]

ATT(i): the expected overall attrition for type "i" aircraft

Subscripts Used:

a: APOD

i: a specific aircraft type, range is from i to I.

For this model i ranges from 1 to 6 with the following aircraft:

1= C-5B Galaxy

2= C-17

3= C-141B Starlifter

4= CRAF Boeing 747(cargo type)

5= CRAF McDonnell Douglas DC-8(personnel type)

6= C-130D/E Hercules

j: a specific type of airlift mission, range is from 1 to J. For this model j ranges from 1 to 4.

1= Delivery to the APOD (Intertheater)

2= Direct delivery to the front (Intertheater)

3= Airdrop missions (Intertheater)

4= Intratheater missions

k: a specific cargo type, range is from 1 to K. For this model k ranges from 1 to 4.

1= outsize

2= oversize

3= bulk(including supplies)

4= personnel

l: a specific period, range is from 1 to L. For this model l ranges from 1 to 4, with each period covering 5 days for 20 days total.

m: the mode of delivery, range is from 1 to M. For this model m ranges from 1 to 3.

1= delivery to the front

2= delivery to the APOD and remain at the APOD

or move to the front on their own

3= delivery to the APOD and move to the front via intratheater airlift.

y: a specific unit type, range is from 1 to Y. For this model range is from 1 to 8.

1= Airborne units of the 82nd Airborne Division

2= HQ units from the 82nd

3= Air Assault Units(AH-64 equipped)

4= Artillery units(155mm)

5= Mechanized battallion (M-1's)

6= Aircraft Fighter Squadron (F-16's)

7= Medium truck companies

8= USAF Airlift Control Elements (ALCE)

Parameters Used:

ATTRIT(i,j): the attrition rate for type "i" aircraft flying on mission type "j".

ADC(i): airdrop capability of aircraft type "i" in number of standard pallets.

AT(y): anti-tank capability of type "y" unit expressed in equivalent TOEs.

CI(y): the combat indicator: 1= combat unit, -1= combat support, 0= neither combat or combat support unit.

CPI(1): the combat power index for arriving at the front in period 1.

CPLOST(i): the expected combat power lost for every aircraft load of type "i" lost due to combat

DAPDFT: the distance between the APOD and the FOL.

DUSAPD: the distance between the U.S. and the APOD.

DUSFRT: the distance between the U.S. and the FOL.

EAS(i,k): the standardizing factor for the material handling equipment required to unload aircraft type "i" with type "k" cargo.

FLT(y): the front line trace capability of a type "y" unit.

FP(y): the firepower capability of type "y" unit.

GAT(l): the desired goal for anti-tank power by period "l".

GFLT(l): the desired goal for front line trace by period "l".

GCP(l): the desired goal for combat power by period "l".

GT(i): the authorized ground time for type "i" aircraft to offload/onload and refuel if required (hours/aircraft).

Ha: the material handling equipment capacity in pallet equivalents existing at the APOD prior to the deployment (pallets/day).

Hf: the material handling equipment capacity in pallet equivalents existing at the FOL prior to the deployment (pallets/day).

INTER(i): the number of days required for aircraft type "i" between intertheater missions (days).

INTRA(i): the number of days required for aircraft

type "i" between intratheater missions (days).

Cargo(i,k): the cargo capacity of type "i" aircraft with type "k" cargo (tons/ac).

MHE(i): the pallet capacity of type "i" aircraft for airland missions (463L pallets).

My: the movement time required by a unit to go from the APOD to the FOL by ground transportation, rounded to the nearest integer (periods).

L: the number of periods considered in the model. For this model four periods are considered, each period lasting five days.

NPAL: the number of pallet equivalents that an ALCE unit can manage in one day (pallets/day).

NPRKA(i): the number of type "i" aircraft that can simultaneously park on the APOD ramp (aircraft).

NPRKF(i): the number of type "i" aircraft that can simultaneously park on the FOL ramp (aircraft).

NPK(y): the number of type "y" unit fighter's that can simultaneously park on the APOD ramp (ac).

NTAC(y): the number of fighter aircraft assigned to type "y" unit (aircraft/unit).

NUM(i,1): the number of type "i" aircraft available for the first period and committed for this specific operation (aircraft).

NUM(i,l): the number of aircraft available for period "l"(after attrition is considered).

PL: the number of days in one period. For this model

PL = 5 days.

RC: the number of pallets the Army Riggers can prepare in one day (pallets/day).

SPD(i): the no-wind cruise speed of type "i" aircraft

TON(y,k): the amount of tonnage that must be moved for type "y" unit of type "k" cargo (tons/ac).

TON(y,3): the supply consumption rate of type "y" unit (tons/unit/day).

TMC(y): the ton-mileage capability for the transportation of supplies to the front for type "y" units (ton/miles/unit/day).

TVL(y): the distance that unit type "y" can travel in one day.

Unit(y): the total number of type "y" units available

UTE(i,j): the utilization rate of type "i" aircraft on a type "j" mission (hours/day).

Objective

When attempting to maximize the amount of combat power delivered to a theater commander, it is evident that the scenario will affect which types of units will be more effective. The three criteria used in this model to quantify combat power are anti-tank capability, firepower, and defense frontage (front line trace or FLT) (7:52). All three measures of effectiveness contribute to total combat power delivered, but each would possess a certain

advantage given different scenario limitations. Otherwise, one could state that the combat attributes were "mutually reinforcing" and attempt to maximize all three variables regardless of the types of units deployed or scenario parameters. In reality, if the scenario involves a large mechanized enemy force, the deployed units' ability to project a strong anti-tank capability may well override its ability to defend a given amount of ground. These real-world constraints dictate a model which allows the user the opportunity to stress the importance of one measure of combat power over another.

Two of these three measures of combat power are independent of time, whereas the third is very time dependent. The amount of anti-tank capability and defensive frontage are independent of time. The third measure of combat power, firepower, emphasizes a point discussed in the unclassified portion of the Congressionally Mandated Mobility Study (8:30). This concept stressed the importance of timely delivery and stated that the delivery of a small force to the correct location can be up to six times as effective as the same force delivered late. The time independent goals of anti-tank capability (AT) and defensive frontage (FLT) will be discussed first. Anti-tank capability is represented by using the multiplier AT(y) for each of "y" units. For time period 1', total anti-tank potential is described with the following equation:

$$\sum_{y=1}^{Y'} \sum_{m=1}^{3} \sum_{l=1}^{l'} AT(y)*u(y,m,l)$$

This general expression must now be broken down into the different methods used to deliver units to the front. As discussed in an earlier section of this chapter, m=1 is used to represent direct delivery to the front; m=2 represents self-propelled units which arrive at the APOD then travel to the forward operating location on their own. The capability of these units should not be considered until the units actually arrive at the front and the units' assets are available to the area commander. To consider the delay, the variable "My" is defined as the number of periods required by the unit to travel from APOD to FOL. The constraint for m=2 now becomes:

$$\sum_{y=1}^{Y'} \sum_{l=1}^{l'-My} AT(y) * u(y,2,l)$$

The representation for units requiring intratheater airlift is (m=3). As stated by Cooke, the time required to go from APOD to FOL is not considered because the enroute time is significantly less than the time for one period to elapse. The expression becomes:

$$\sum_{y=1}^{Y'} \sum_{l=1}^{l'} AT(y) * u(y,3,l)$$

The last area to be considered consists of units assigned to the APOD that possess an anti-tank potential. The next equation considers units at the APOD that contribute to the total anti-tank power:

$$\sum_{y=Y'+1}^{Y} \sum_{l=1}^{l'} AT(y) * u(y,2,l)$$

Summing the possible sources of anti-tank power for period l' produces the following equation:

$$\sum_{y=1}^{Y} \sum_{l=1}^{l'} \sum_{m=1,3} AT(y) * u(y,m,l) + \sum_{y=1}^{Y'} \sum_{l=1}^{l'-My} AT(y) * u(y,2,l) +$$

$$\sum_{y=Y'+1}^{Y} \sum_{l=1}^{l'} AT(y) * u(y,2,l) \geq GAT(l') \qquad (1)$$

The first summation encompasses direct (m=1) and intratheater delivery (m=3), while the second term (m=2) includes units that reach the front via ground transportation, and the third summation includes units assigned to the APOD. (7:53;33:Chap 3,8).

Defensive frontage (FLT) is developed in a similar manner. In this case, the multiplier FLT(y) is used to represent the amount of defensive frontage capable of being defended by a specific unit. The development of the constraint follows the same format as the previous constraint and results in the following equation for period l':

$$\sum_{y=1}^{Y'} \sum_{l=1}^{l'} \sum_{m=1,3} FLT(y) * u(y,m,l) + \sum_{y=1}^{Y'} \sum_{l=1}^{l'-My} FLT(y) * u(y,2,l)$$

$$+ \sum_{y=Y'+1}^{Y} \sum_{l=1}^{l'} FLT(y) * u(y,2,l) \geq GFLT(l') \qquad (2)$$

The goal or constraint for this equation, GFLT(l'), represents the defensive frontage required for period l'. These first two measures of combat power are independent of time, whereas the contribution of this third measure of power is directly related to the time that the combat power reaches the objective area. This constraint uses the time-dependent multiplier CPI(l), which represents the combat power index. The index decreases over time placing a higher value on units reaching the objective area early in the conflict. The term FP(y) is the amount of firepower contributed by unit y. The expression for this equation is as follows:

$$\sum_{y=1}^{Y'} \sum_{l=1}^{l'} \sum_{m=1,3} CPI(l) * FP(y) * u(y,m,l) +$$

$$\sum_{y=1}^{Y'} \sum_{l=1}^{l'-My} CPI(l + My) * FP(y) * u(y,2,l) +$$

$$\sum_{y=Y'+1}^{Y} \sum_{l=1}^{l'} CPI(l) * FP(y) * u(y,2,l) \geq GFP(l') \qquad (3)$$

The first summation considers the firepower delivered to the front, the second summation considers firepower delivered to the APOD and transported to the front, and the third summation considers firepower available at the

APOD. The second summation includes the requirement to deliver the men and equipment to the objective area before the combat power index is applicable. The third summation includes the firepower deployed to the APOD which is not to be transported to the front. (7:55;33:Chap.3,9-10).

Any of these three measures of effectiveness (MOE's) can be used interchangeably as the objective function based upon the requirements of the scenario. The other two equations would then be entered in the constraint matrix. These three equations represent the combat power capability of the units to be airlifted during the contingency. Only the equation on firepower delivered is time-dependent, and that concept is demonstrated by using the CPI(1) time delay. The first two equations are not dependent upon time, and measure combat power based upon the unit type and the quantity of that unit delivered to the objective area.

An examination of the three equations reveals a consistent relative ranking of the units. Regardless of whether anti-tank power, defensive frontage, or fire power is maximized, the same units will be delivered to the objective area. Units of type y=4 possess the largest percentage of fire power, defensive frontage, and anti-tank power. The relative relationship between the units is preserved with each of the three possible goals. By simplifying the scenario so the objective is to maximize

fire power, the other two measures of effectiveness can be set as constraints. As long as these two constraints are set at a feasible level, the objective function of maximizing fire power is left relatively unaffected by the setting of defensive frontage or anti-tank capability levels.

For the purposes of this scenario, the goal is to maximize fire power delivered to the theater commander, and the two other measures of effectiveness can be set as greater than or equal to constraints. The objective function equation becomes:

MAXIMIZE:

$$\sum_{y=1}^{Y'} \sum_{l=1}^{l'} \sum_{m=1,3} CPI(l) * FP(y) * u(y,m,l) +$$

$$\sum_{y=1}^{Y'} \sum_{l=1}^{l'-My} CPI(l + My) * FP(y) * u(y,2,l) +$$

$$\sum_{y=Y'+1}^{Y} \sum_{l=1}^{l'} CPI(l)*FP(y)*u(y,2,l) - \sum_{i=1}^{I} CPLOST(i)*ATT(i) \qquad (4)$$

The next section of this chapter discusses the aircraft, cargo, and airfield restrictions that constrain the maximization of combat power delivered to the field commanders. This section also further defines the terms CPLOST(i) and ATT(i) in the equation above.

Formulation of Constraints

The quantity of combat power delivered to an objective area by air is directly affected by the characteristics of the units to be deployed and the capabilities of the aircraft involved in the airlift. Each of the constraints to follow will be discussed in detail in order to examine the influence that specific constraint has upon the maximization of combat power delivered to the theater commanders over a given period of time. The equations were formulated largely by Cooke, updated by Tate, and changes were made here when necessary to reflect conceptual differences or changes in the data base.

Aircraft Limitations

The time required to move units from the CONUS to the objective area is heavily influenced by the total number of aircraft dedicated to the deployment and the ability of each aircraft type to fly at a sustained flying hour level. The first constraint deals with the number of aircraft sorties as determined by the number of aircraft available and the time required for an aircraft to fly a complete mission. A complete mission involves departing home station, flying the required mission legs, and returning to home station after standard ground times, and expected maintenance down times, ready for another home

station departure. The second constraint determines the expected number of aircraft sorties capable of being generated by each aircraft type (UTE rate), based upon the number of hours each aircraft can be expected to fly.

Aircraft Sorties. The first constraint determines the number of aircraft sorties, by aircraft type, based upon the number of aircraft expected to be available for the deployment of the combat units. The total number of sorties is also dependent upon the time required for an aircraft to depart home station, fly the proposed routing with standard ground times and expected maintenance down times, and return to home station ready for another mission. The range and speed of each aircraft differ slightly, and for this specific scenario, data was obtained from MACR55-141 and HQ MAC/XOSS (12). HQ MAC provided the estimated distances to the various bases and forecasted wind factors for the proposed routing. Estimated ground speeds of the various aircraft were determined based upon true airspeeds and wind factors for each mission leg. Using the estimated distances and groundspeeds, an enroute time was determined for each leg of the mission. These estimated enroute times were added together and combined with the estimated maintenance downtimes and scheduled ground times for each aircraft type to determine a total time required to be ready for another mission.

Cooke assumed overflow between days because his period was equal to one day, yet the mathematical formulation used period lengths of five days. Using a similar time frame of four linked five-day periods will adequately represent the time period considered most critical for testing the effectiveness of airlift in delivering combat power to a given objective area.

Thus far two factors have been discussed which affect the total number of aircraft sorties available for the airlift, the time required for an aircraft to transit the proposed route structure and the number of aircraft expected to be available for the operation. A third factor affecting sortie generation is possible aircraft attrition. Attrition would be related to the aircraft and mission type. Attrition for a specific aircraft and mission type is a subjective determination with a number of possible complicating factors. While some theorists believe a direct delivery aircraft, such as the C-17, is more susceptible to enemy fire because it flies closer to the actual battle area, other sources believe that direct delivery disperses the airlift force over a wider area, thereby, enhancing the survivability of all airlift aircraft operating in a threat environment. (30:33). The C-17 will possess increased capability to airland and airdrop men and supplies directly to the objective area while minimizing the threat to the aircraft, but the

threat still exists. A further discussion of attrition follows in the development of the scenario. Attrition variables are included in the model and the coefficients have been determined subjectively. The purpose of including attrition in the model is not to validate the attrition values but to examine the effects of attrition on the delivery of combat power. Attrition values will no doubt be debatable, but hopefully the relative scaling of the attrition values are reasonable estimates of potential attrition in a low threat environment. The mathematical equation depicts the negative effect of attrition on sorties available. The attrition of the number of aircraft available for a given period of time is developed by first setting the expected number of aircraft available in the first period with the variable NUM(i,1). Aircraft type "i" begins the operation in subsequent periods with a given number of aircraft available from period one minus any attrition of aircraft. The relationship is NUM(i,l) = NUM(i,1) - Attrition. Attrition depends upon aircraft type and mission type and is formulated as:

$$\text{Attrition} = \sum_{l=1}^{l'-1} \sum_{j=1}^{J} \sum_{k=1}^{K} \text{Attrit}(i,j) * x(i,j,k,l')$$

Since we are dealing with periods, and not days the formulation becomes:

For l=1: NUM(i,1)*Pl = NUM(i,1)*PL

For l=2,3,4: $NUM(i,l)*Pl=NUM(i,l)*Pl$

$$- \sum_{l=1}^{l'-1} \sum_{j=1}^{J} \sum_{k=1}^{K} Attrit(i,j)*PL * x(i,j,k,l')$$

By rearranging the terms, the final equation to determine the number of sorties available during the deployment phase is:

For l= 1:

$$Inter(i)* \sum_{j=1}^{3} \sum_{k=1}^{K} x(i,j,k,l) + Intra(i)* \sum_{k=1}^{K} x(i,4,k,l) \leq Num(i,l)*PL$$

For l= 2,3,4:

$$Inter(i)* \sum_{j=1}^{3} \sum_{k=1}^{K} x(i,j,k,l) + Intra(i)* \sum_{k=1}^{K} x(i,4,k,l) + \sum_{l=1}^{l'-1} \sum_{j=1}^{J} \sum_{k=1}^{K} Attrit(i,j)*x(i,j,k,l') \leq Num(i,l)*PL \quad (5)$$

There will be one constraint for each aircraft type per period, or a total of (I*L) equations. (33:Chap 3,11)

<u>UTE Rate</u>. The second set of constraints related to aircraft limitations consider the expected usage rate of the aircraft. The expected utilization (UTE) rate is a commonly used method of measuring the airlift capability of an aircraft. UTE rate is computed by dividing the total programmed flying hours for a given aircraft type

(TPFH) by the primary aircraft authorized (PAA) and dividing that number by 360 to get an expected daily usage rate (12). An approximation of the expected surge capability UTE rate, which is higher than the standard "how much are we flying today rate" is used for this scenario. The average number of sorties each particular aircraft type can be expected to fly is depicted as follows:

$$[PL*UTE(i,j)*Num(i)*SPD(i)]/(2*DUSAPD) = \text{aircraft/period}$$

The period length * the UTE rate * number of aircraft available * speed of the aircraft divided by two * the distance from the United States to the APOD = required aircraft/period. As noted by Cooke and Tate, the two in the denominator is used to denote the round trip distance involved in the airlift. The terms can be rearranged to develop the following constraint:

$$\sum_{j=1}^{J} \sum_{k=1}^{K} [1/UTE(i,j)]*x(i,j,k,l') \leq [PL*Num(i)*SPD(i)]/(2*DUSAPD)$$

A standardizing factor was developed by Cooke to convert aircraft of the same type, flying different missions to an equivalent number of sorties for a respective mission. The ratio [(distance from other)/(distance from US to APOD)] shows that there are more sorties available for intratheater (j=4) missions than for intertheater missions (j=1,2,3). It should also

be noted that any aircraft used strictly for intratheater missions, such as the C-130, should be considered separately. The number of sorties they can generate are dependent upon the number of aircraft available and the distance from the APOD to the FOL.

The general expression for the constraint for period 1' and for aircraft types 1 through 5 is as follows:

$$\sum_{k=1}^{K} \sum_{j=2,3} 1/UTE(i,j)* \frac{DUSFRT}{DUSAPD} *x(i,j,k,1') + \sum_{k=1}^{K} 1/UTE(i,1)* x(i,1,k,1') + \sum_{k=1}^{K} 1/UTE(i,4)* \frac{DAPDFT}{DUSAPD} *x(i,4,k,1') \leq PL*Num(i)*SPD(i)/(2*DUSAPD) \qquad (6)$$

The first summation accounts for the direct (j=2) and airborne (j=3) missions, the second summation accounts for intertheater missions, and the third summation accounts for aircraft capable of also flying intratheater missions (j=4). The C-130 sorties, used strictly for intratheater missions, use the following equation:

$$\sum_{k=1}^{K} 1/UTE(4,4)*x(6,4,k,1') \leq PL*Num(6)*SPD(i)/2*DAPDFT \qquad (7)$$

There is one constraint for each type of aircraft per period for a total of (I*L) constraints (7:59-60;33: Chap3,11-13).

Airport Limitations

Airport facilities provide three additional constraints on strategic mobility. The first constraint is on the capacity of the APOD ramp to park available aircraft, the second is the capacity of the FOL ramp to park available aircraft, and the third constraint is the capability of the material handling equipment (MHE) to offload arriving aircraft and onload aircraft for departure. As in previous thesis efforts on this subject, a linear relationship is assumed between the area required to park different aircraft types. As an example, if the ramp space is sufficient to park 30 C-130's or 20 C-141's, it is also sufficient to park 15 C-130's and 10 C-141's or any other convex combination. As pointed out by Cooke, such approximations are sufficient for all but the smallest of airfields, and small airfields are not usually selected as main operating bases because of ramp space limitation (7:61).

Parking space per unit time for each aircraft type is used as the parameter to measure available ramp space. Using the scheduled ground time [GT(i)] and the number of aircraft of a given type that can park at the APOD [NPRK(i)], it is possible to determine how many sorties of a given aircraft type can be generated through the APOD in a single period. The parking space is constrained as follows:

$$\sum_{i=1}^{I} \sum_{j=1,4} \sum_{k=1}^{K} x(i,j,k,l) * GT(i) / [PL*NPRKA(i)*24] \leq 1$$

When the left hand side of the constraint equals one, ramp space on the field is completely saturated. The number of aircraft for each type that can fit on the ramp was obtained from a HQ MAC maximum on the ground listing (MOG). The listing includes several hundred bases currently surveyed by MAC and published by HQ MAC/DEP.

The possibility exists that fighter aircraft may be assigned to the APOD. If that were the case, they would also require parking spots and the constraint would now be:

$$\sum_{i=1}^{I} \sum_{j=1,4} \sum_{k=1}^{K} x(i,j,k,l) * GT(i)/[PL*NPRKA(i)*24] + \sum_{y=y'+1}^{Y} U(y,2,l)*NTAC(y)/PL*NPK(y) \leq 1 \qquad (8)$$

The first summation includes aircraft flying directly to the APOD or flying intratheater missions between the APOD and FOL. The second summation includes any fighter units stationed at the APOD. There is one constraint per period for a total of (L) constraints (7:62-63).

The second constraint is very similar to the first, except it determines the parking capacity at the FOL. It was not included in previous efforts but should be added for completeness. The mathematical formulation is as follows:

$$\sum_{i=2,3,6} \sum_{j=2}^{4} \sum_{k=1}^{K} x(i,j,k,l)*GT(i)/[PL*NPRKF(i)*24] \leq 1 \quad (9)$$

The equation only considers C-17, C-141 and C-130's because the other aircraft types will not transit the front. As in the previous equation, there should be (L) constraints.

The third constraint determines the airfield's capability to offload and onload equipment at the APOD and offload equipment at the front. The time required to upload or download an aircraft is dependent upon the number of pallets an aircraft can carry and the type of cargo to be moved. The number of pallets (NPAL) that the various aircraft in this scenario can hold is listed in Table 2. The ease with which each aircraft can be loaded or unloaded [EAS(i,k)] is a subjective determination based upon the aircraft type, cargo type, and sophistication of loading equipment (k-loaders vs fork lifts). If all the aircraft can be loaded with the same degree of difficulty, all aircraft will be given an index of one. The arriving aircraft can expect to be downloaded by the equipment (K-loaders, forklifts, etc.) that are currently available at

the APOD (Ha) and the front (Hf). Once an ALCE unit is in place, it will provide additional offloading capability. A multiplier is used to approximate the additional assistance gained from any arriving ALCE unit. For this model a multiplier of 0.5 is selected as a representative level of improvement expected by the arrival of ALCE units. The constraint must also consider the restriction that the number of aircraft moving through the APOD or FOL cannot be greater than the capabilities of the airfield MHE to upload or download them. The constraints are as follows:

<u>APOD</u>:

$$\sum_{i=1}^{I} \sum_{j=1,4} \sum_{k=1}^{K} EAS(i,k)*MHE(i) * x(i,j,k,l')$$

$$- \sum_{l=1}^{l'-1} NPAL*PL*u(8,2,l) - 0.5*NPAL*PL*u(8,2,l') \leq Ha*PL \quad (10)$$

<u>For the Front</u>:

$$\sum_{i=2,3,6} \sum_{j=2,4} \sum_{k=1}^{K} EAS(i,k) * MHE(i) * x(i,j,k,l')$$

$$- \sum_{l=1}^{l'-My} \sum_{m=1,3} NPAL*PL*u(8,m,l) \leq Hf*PL \quad (11)$$

There is one constraint per period for both the APOD and the FOL, or (2*L) constraints, (7:61-64;33:Chap 3, 13-17).

Unit Limitations

This constraint encompasses the restriction that the number of units shipped must be equal to or less than the total number of units available. The constraint is expressed as:

$$\sum_{m=1}^{3} \sum_{l=1}^{L} u(y,m,l) \leq \mathrm{Unit}(y) \qquad (12)$$

There is one constraint for each type of unit, for a total of Y constraints. (7:64;33:Chap 3,17)

Shipment of Units

Units can be delivered to the APOD using three primary methods, 1) via the APOD, 2) via direct delivery, and 3) aerial delivery.

The first section discusses the delivery of units to the APOD. When discussing the delivery of units to a given area, the entire unit must be delivered before the unit is considered "closed". Each unit must be delivered in its entirety. Outsized cargo, oversized cargo, bulk cargo, and personnel must all be delivered before the unit is considered ready for its combat or support mission. Excess cargo [A(k,l)] or early cargo [A(K,l-1)] also needs

to be considered in formulating the constraint. The complete formulation is as follows:

$$\sum_{i=1}^{I} cargo(i,k) * x(i,1,k,l) - \sum_{y=1}^{Y} \sum_{m=2}^{3} TON(y,k) * u(y,m,l) + A(k,l-1) - A(k,l) = 0 \quad (13)$$

To insure that cargo moved to the APOD is delivered to the front, the following constraint is added:

$$\sum_{i=1}^{I} cargo(i,k) * x(i,4,k,l) - \sum_{y=1}^{Y'} TON(y,k)*u(y,3,l) + P(k,l-1) - P(k,l) = 0 \quad (14)$$

There are two constraints for every cargo type and period for a total of (K*L) constraints.(7:65-66;33:chap 3,17-18)

The second section discusses the delivery of units to the front via direct delivery. The mathematical expression is very similar to the APOD equation with the exception that only aircraft flying direct missions will be considered:

$$\sum_{i=1}^{I} cargo(i,k) * x(i,2,k,l) - \sum_{y=1}^{Y} TON(y,k)*u(y,1,l) + P(k,l-1) - P(k,l) = 0 \quad (15)$$

Again, there is one constraint for each cargo type and period for a total of (K*L) constraints. (7:66;33:chap 3, 18)

The last method to be discussed is the use of airdrop missions to deliver the men and supplies. An indicator variable, AB(y) = 1 or 0, is used to indicate whether the unit is airdrop capable or not. The only aircraft considered are those aircraft capable of intertheater airdrop missions, the C-17 and the C-141. While the C-130 is also airdrop capable, it is normally used for intratheater missions (no air refueling capability), and intratheater airdrop missions are not considered in this scenario. The expression is:

$$\sum_{i=2,3} \text{cargo}(i,k)*x(i,3,k,l) - \sum_{y=1}^{Y'} AB(y)*TON(y,k)*u(y,1,l) + P(k,l-1) - P(k,l) = 0 \quad (16)$$

As stated before, there is one constraint for each period and cargo type, or (L*K) constraints. (7:66;33:Chap 3, 18-19).

Shipment of Supplies

Supplies are considered bulk items (K=3), and once a unit is delivered to the objective location, supplies must be kept at a stabilized level in order to sustain the unit's operation. The input of supplies must exceed the

consumption of supplies for all the deployed units. The shipment of supplies will be discussed in three separate sections. The first section discusses the requirement that supplies reach the units at the front, and the last section discusses the need to adequately supply units based at the APOD.

This section discusses the need to insure adequate supplies reach the front. An assumption for this constraint is that any unit not based at the APOD is based at the FOL and will require its supplies to reach the front. Units such as the fighter squadron and transportation units are expected to require APOD supplies; whereas, the other units in the model will require supplies at the FOL. It is assumed that units arriving in a given period have enough supplies to sustain the unit for the remainder of the period. With this assumption, the requirement is to insure adequate supplies are delivered to units arriving in previous periods only. As stated in Tate's thesis, supplies can be delivered to the front via direct delivery, airborne delivery, transportation via intratheater airlift, and overland shipment by truck. Leaving the truck shipments to be discussed later, the total amount of supplies delivered by airlift is found by adding the supplies delivered by direct delivery, ($j=2$), airborne delivery ($j=3$), and intratheater transshipment ($j=4$). The supplies transported are represented by:

$$\sum_{i=1}^{I} \sum_{j=2}^{4} cargo(i,3) * x(i,j,3,l')$$

For those units being shipped by truck, it is necessary to consider the time required to travel from the APOD to the FOL and the capability of the unit to move supplies. The parameter TRP(y) is used to designate the capability of a unit to move supplies. This parameter is applicable to units such as a truck company which can be expected to deliver supplies from the APOD to the FOL. The constraint expressing the number of tons a unit can move in one period is:

$$TRP(y)*PL/2*DAPDFT$$

The two in the denominator reflects the round trip distance from the APOD to the front. The total tonnage shipped between the APOD and FOL by truck is as follows:

$$\sum_{y=1}^{Y} \sum_{m=1}^{3} \sum_{l=1}^{l'-My} TRP(y)*PL/2*DAPDFT]*u(y,m,l)$$

The next equation combines the expressions for delivering supplies to the front and subtracting the consumption of the units previously delivered to the front. The equation is:

$$\sum_{i=1}^{I} \sum_{j=2}^{4} cargo(i,3)*x(i,j,3,l') + \sum_{y=1}^{Y} \sum_{m=1}^{3} \sum_{l=1}^{l'-My} [TRP(y)*PL/$$

$$2*DAPDFT]*u(y,m,l) - \sum_{y=1}^{Y'} \sum_{m=1}^{3} \sum_{l=1}^{l'-1} PL*TON(y,3)*u(y,m,l) \geq 0$$

An additional constraint must be added to insure that the aircraft and trucks do not attempt to deliver supplies that are not available at the APOD. Letting S1 equal the supplies transshipped during period l and formulating the constraint such that the amount of supplies delivered by airlift plus the supplies delivered by truck are greater than or equal to the supplies delivered during the period, the expression for period l' becomes:

$$\sum_{i=1}^{I} cargo(i,3)*x(i,4,3,l') + \sum_{y=1}^{Y} \sum_{m=1}^{3} \sum_{l=1}^{l'-My} [TRP(y)*PL]/$$

$$(2*DAPDFT) * u(y,m,l) \geq S1$$

The final equation, letting the parameter TON(y,3) represent the daily consumption rate for unit "y", PS(3,l') represent any surplus supplies shipped at the end of the l'th period, and PS(3,l'-1) represent the surplus from previous periods, is:

$$\sum_{i=1}^{I} \sum_{j=1}^{4} cargo(i,3) * x(i,j,3,l') + \sum_{y=1}^{Y} \sum_{m=1}^{3} \sum_{l=1}^{l'-My} [TRP(y) * PL/$$

$$2*DAPDFT] * u(y,m,l) - \sum_{y=1}^{Y'} \sum_{m=1}^{3} \sum_{l=1}^{l'-1} PL*TON(y,3) * u(y,m,l)$$

$$+ S1 - PS(3,l'-1) + PS(3,l) = 0 \qquad (17)$$

There are two constraints per period or a total of (2 *L) constraints (7:68-9;33:Chap.3,22-23).

The next equation deals with the constraint of supplies for the APOD. As in the previous cases, the supplies reaching the APOD can be no less than the requirements of the deployed units. All the supplies delivered to the APOD are either consumed at the APOD, delivered by intratheater airlift to the front, or shipped by trucks to the front. The supply constraint is the sum of the supplies delivered to the APOD for this period plus any surplus supplies from the previous period, minus the amount of supplies either consumed at the APOD, transshipped to the front, and any excess supplies for the period. The equation includes the same S1 variables which equates to the amount of supplies delivered from the APOD to the front for a given period. This scenario assumes that the battalion headquarters, F-16 squadron, and the truck company receive their supplies at the APOD. The resulting equation is:

$$\sum_{i=1}^{I} cargo(i,3) * x(i,1,3,l') + A(3,l'-1) - S1 - A(3,l')$$

$$- \sum_{y=Y'+1}^{Y} \sum_{l=1}^{l'-1} PL * TON(y,3) * u(y,2,l) = 0 \qquad (18)$$

For clarification, the first Y' units are deployed to the front while the remainder are deployed at the APOD. There is one constraint per period for a total of (L) constraints. (7:70-71;33:Chap 3,23-24).

Unit Linkage

This section deals with the relationship between distinct but related unit types. Certain units require that a minimum number of other related units be deployed if they are to be deployed. This relationship is examined in the model by requiring a linkage between the number of combat units and combat support units, such as the link between a battalion and battalion headquarters. These linkages establish certain ceilings where only a given amount of one unit can be deployed unless another related unit increases its deployed forces. The inverse relationship exists for floors of certain units whereby particular units must be deployed in order for some distinct but related unit to increase its resources.

As an example of a ceiling, the following equation requires a minimum number of combat support units to be deployed to backup the combat units. A certain ratio

is established between combat and combat-support units. The next equation uses a multiplier [CI(y)] to distinguish between combat, combat support, and other units. The multiplier is given the value of 1,-1, and 0 respectively. An "other" unit may include an ALCE unit which is neither a combat unit or combat support unit. The formulation is as follows:

$$\sum_{y=1}^{Y} \sum_{m=1}^{3} \sum_{l=1}^{L} CI(y) * u(y,m,l) \geq 0 \qquad (19)$$

This equation adds one constraint to the problem. (7:71, 33:Chap 3,24-25).

A floor is another typical unit linkage which requires deployment of a minimum of a certain type of unit given that other units are already deployed. This relationship exists between combat units and combat support units, such as a battalion headquarters. One method of linking a one to five ratio between HQ units to battalion units is:

$$\text{combat unit} - 5*\text{Hq unit} \leq 0 \qquad (20)$$

This formulation forces a HQ unit to precede the combat battalion in the initial deployment, which is neither desirable nor realistic. The following expression: combat-3HQ $\leq$ 2, allows up to two combat units

to deploy before a HQ unit is required and allows a reasonable number of combat units to deploy for every HQ unit deployed. This equation requires an additional constraint for each linkage between combat units and other types of units in the scenario. (7:72;33:Chap 3,25).

Unit Abilities

This constraint examines the limitation on airdrop capability imposed by the ability of rigging personnel to properly configure the units' assets into pallets capable of being airdropped. Riggers can only rig so many pallets in a given time period, and this expression can be expressed mathematically as:

$$\sum_{i=1}^{I} ADC(i,3)*x(i,3,3,l) - P(3,l-1) + P(3,l) = RC*PL \qquad (21)$$

The variable ADC(i,3) refers to the number of pallets an aircraft (i) can hold in the airdrop configuration, and RC refers to the rigging capability of the army riggers in pallets per day. The P(3,l-1) and P(3,l) represent the previous periods surpluses and the present periods surpluses. There is one constraint per period or (L) constraints (7:73;33:Chap 3,25-26).

Summary of Goals and Constraints

The purpose of this chapter is to examine the mathematical equations used to derive the interrelationships between aircraft capabilities, cargo characteristics, and airfield restrictions. The majority of the equations were derived by Cooke, and with only minor changes, the variable names were determined by Tate. The data base that is applied to these mathematical equations varies from previous efforts in numerous areas and will be discussed in the next chapter. This chapter concludes with a summary of the objective and constraints as follows:

OBJECTIVE:

Fire Power Capability

MAXIMIZE:

$$\sum_{y=1}^{Y'} \sum_{l=1}^{l'} \sum_{m=1,3} CPI(l)*FP(y)*u(y,m,l) + \sum_{y=1}^{Y'} \sum_{l=1}^{l'-My} CPI(l+My)*FP(y)* u(y,2,l) + \sum_{y=Y'+1}^{Y} \sum_{l=1}^{l'} CPI(l)*FP(y)*u(y,2,l) - \sum_{i=1}^{I} CPLOST(i) * ATT(i) \quad (4)$$

SUBJECT TO THE CONSTRAINTS:

Anti-Tank Capability

$$\sum_{y=1}^{Y'} \sum_{l=1}^{l'} \sum_{m=1,3} AT(y)*u(y,m,l) + \sum_{y=1}^{Y'} \sum_{l=1}^{l'-My} AT(y)*u(y,2,l) +$$

$$\sum_{y=Y'+1}^{Y} \sum_{l=1}^{l'} AT(y)*u(y,2,l) \geq \text{Anti-tank goal} \quad (1)$$

Front Line Trace Capability

$$\sum_{y=1}^{Y'} \sum_{l=1}^{l'} \sum_{m=1,3} FLT(y)*u(y,m,l) + \sum_{y=1}^{Y'} \sum_{l=1}^{l'-My} FLT(y)*u(y,2,l) +$$

$$\sum_{y=Y'+1}^{Y} \sum_{l=1}^{l'} FLT(y)*u(y,2,l) \geq \text{Front Line Trace Goal} \quad . (2)$$

Aircraft Sortie Generation Capability (One for each aircraft type and period)

$$Inter(i)*\sum_{j=1}^{3} \sum_{k=1}^{K} x(i,j,k,l) + Intra(i)*\sum_{k=1}^{K} x(i,4,k,l) +$$

$$\sum_{l=1}^{l'-1} \sum_{j=1}^{J} \sum_{k=1}^{K} Attrit(i,j)*x(i,j,k,l') \leq Num(i,l)*PL \quad (5)$$

Aircraft Utilization (UTE) Rate (One for each aircraft type and period)

$$\sum_{k=1}^{K} \sum_{j=2,3} 1/UTE(i,j)* \frac{DUSFRT}{DUSAPD} *x(i,j,k,l') + \sum_{k=1}^{K} 1/UTE(i,1)*$$

$$x(i,1,k,l') + \sum_{k=1}^{K} 1/UTE(i,4)* \frac{DAPDFT}{DUSAPD} *x(i,4,k,l')$$

$$\leq PL*Num(i)*SPD(i)/(2*DUSAPD) \qquad (6)$$

Intratheater (UTE) Rate (C-130)

$$\sum_{k=1}^{K} 1/UTE(4,4)*x(i,4,k,l') \leq PL*Num(i)*SPD(i)/2*DAPDFT \qquad (7)$$

APOD Ramp Space (One for each period)

$$\sum_{i=1}^{I} \sum_{j=1,4} \sum_{k=1}^{K} x(i,j,k,l) * GT(i)/[PL*NPRKA(i)*24] +$$

$$\sum_{y=y'+1}^{Y} u(y,2,l)*NTAC(y)/PL*NPK(y) \leq 1 \qquad (8)$$

FOL Ramp Space (One for each period)

$$\sum_{i=2,3,6} \sum_{j=2}^{4} \sum_{k=1}^{K} x(i,j,k,l)*GT(i)/[PL*NPRKF(i)*24] \leq 1 \qquad (9)$$

APOD Material Handling Equipment (MHE) (One for each period)

$$\sum_{i=1}^{I} \sum_{j=1,4} \sum_{k=1}^{K} EAS(i,k)*MHE(i) * x(i,j,k,l')$$

$$- \sum_{l=1}^{l'-1} NPAL*PL*u(8,2,l) - 0.5*NPAL*PL*u(8,2,l') \leq Ha*PL \qquad (10)$$

FOL Material Handling Equipment (MHE) (One for each period)

$$\sum_{i=2,3,6} \sum_{j=2,4} \sum_{k=1}^{K} EAS(i,k) * MHE(i) * x(i,j,k,l')$$

$$- \sum_{l=1}^{l'-My} \sum_{m=1,3} NPAL*PL*u(8,m,l) \leq Hf * PL \qquad (11)$$

Unit Limitations (One for each unit type)

$$\sum_{m=1}^{3} \sum_{l=1}^{L} u(y,m,l) \leq Unit(y) \qquad (12)$$

Shipment of Units To the APOD (Two for each period and cargo size)

$$\sum_{i=1}^{I} cargo(i,k) * x(i,1,k,;) - \sum_{y=1}^{Y} \sum_{m=2}^{3} TON(y,k) * u(y,m,l)$$

$$+ A(k,l-1) - A(k,l) = 0 \qquad (13)$$

To insure that cargo moved to the APOD is delivered to the front, the following constraint is added:

$$\sum_{i=1}^{I} cargo(i,k) * x(i,4,k,l) - \sum_{y=1}^{Y'} TON(y,k)*u(y,3,l)$$

$$+ P(k,l-1) - P(k,l) = 0 \qquad (14)$$

<u>Shipment of Units via Direct Delivery</u> (One for each period and cargo size)

$$\sum_{i=1}^{I} cargo(i,k) * x(i,2,k,l) - \sum_{y=1}^{Y} TON(y,k)*u(y,1,l)$$
$$+ \quad P(k,l-1) - P(k,l) = 0 \qquad (15)$$

<u>Shipment of Units via Airdrop</u> (One for each period and cargo size)

$$\sum_{i=2,3} cargo(i,k)*x(i,3,k,l) - \sum_{y=1}^{Y'} AB(y)*TON(y,k)*u(y,1,l)$$
$$+ \quad P(k,l-1) - P(k,l) = 0 \qquad (16)$$

<u>Front Line Supplies</u> (One for each period)

$$\sum_{i=1}^{I} \sum_{j=1}^{4} cargo(i,3) * x(i,j,3,l') + \sum_{y=1}^{Y} \sum_{m=1}^{3} \sum_{l=1}^{l'-My} [TRP(y) * PL/$$

$$2*DAPDFT] * u(y,m,l) - \sum_{y=1}^{Y'} \sum_{m=1}^{3} \sum_{l=1}^{l'-1} PL*TON(y,3) * u(y,m,l)$$

$$+ \quad S1 - PS(3,l'-1) + PS(3,l) = 0 \qquad (17)$$

<u>APOD Supplies</u> (One for each period)

$$\sum_{i=1}^{I} cargo(i,3) * x(i,1,3,l') + AS(3,l'-1) - S1 - AS(3,l')$$

$$- \sum_{y=Y'+1}^{Y} \sum_{l=1}^{l'-1} PL * TON(y,3) * u(y,2,l) = 0 \qquad (18)$$

<u>Linkage Between Combat and Combat Support Units</u> (One for the model)

$$\sum_{y=1}^{Y} \sum_{m=1}^{3} \sum_{l=1}^{L} CI(y) * u(y,m,l) \geq 0 \qquad (19)$$

<u>Linkage Between Combat Units and HQ's Units</u> (One for each Headquarters unit)

$$\text{combat unit} - 3 * \text{HQ unit} \leq 2 \qquad (20)$$

Army Pallet Rigging Capability (One for each period)

$$\sum_{i=1}^{I} ADC(i,3)*x(i,3,3,l)-P(3,l-1) + P(3,l) = RC*PL \qquad (21)$$

Using the assumptions discussed in the scenario chapter, the final list of equations produces a model with 168 equations and 288 variables. The linear programming model was constructed using Proc IML to build a 168 by 288 matrix consisting of 48,384 zero elements. The approximately 2400 nonzero elements were inserted using matrix notation, and the matrix was converted to a data set. The right hand side values were constructed as a two variable vector consisting of constraint sign and right hand side value. This vector was merged with the data set to form the objective equation and the 167 constraints. The linear program was executed using the SAS/OR statistical package (28). SAS/OR was also used to execute the required number of runs to build the dependent variable outputs required to construct the proper seven variable experimental design developed by Box and Behnken (3:461). Once the required linear programming runs were completed, the SAS regression package (26) was used to develop the response surface equation. The response surface equation and its applications are discussed in the analysis chapter. SAS/Graph was then used to construct a

variety of three-dimensional plots by varying two of the independent variables while setting the other variables in the model at a constant value (27). The value of the objective function was plotted as it varied with the two selected independent variables, and the result is a visual presentation of the interrelationships existing between the variables. A more complete discussion of the use of these statistical packages and their applications is presented in the analysis chapter.

IV. Scenario Development

Introduction

In an effort to evaluate the usefulness of the model developed in the previous chapter, a scenario centered around the Far East was generated. The theater of operation encompasses the Korean peninsula, and the military units were deployed from the CONUS. More specifically, Kwangju AFB was the APOD and Suwon Airfield was the FOL. The units were flown from Travis AFB to the Far East. The route structure is discussed in further detail in this chapter. The scenario was greatly simplified, and the output of the model is meant to demonstrate the feasibility and usefulness of the model as a tool in determining optimum force structuring, not as the absolute best answer to an actual scenario. Several simplifying assumptions for the scenario are listed next.

Assumptions

The assumptions for this model include:

1) Aircraft will adhere to standard authorized ground times as per the appropriate MAC 55-series regulation.

2) Aircraft are authorized to concurrently refuel while uploading or downloading cargo.

3) The enroute times to transit the system are based upon no air refueling.

4) The majority of the airlift requirements fall under the responsibility of 22nd Air Force, with units from Travis AFB, McChord AFB, Norton AFB, Clark AFB, and Yokota AB.

5) CRAF aircraft, along with respective military aircraft, will use Travis AFB as the upload point and primary point of departure.

6) The required "y" units to be deployed are assumed to be in position and awaiting upload at Travis AFB.

7) All intertheater aircraft will use Travis as a staging point.

8) Aircrews will be staged at all enroute stops except the FOL. While aircraft will spend the scheduled time on the ground, aircrews will be relieved as required to provide a continuous availability of aircrews.

9) C-130's are not involved in intertheater airland or airdrop operations, but, along with C-17's, provide the intratheater airlift support.

10) C-17's and C-141's are the only aircraft capable of direct delivery to the FOL.

11) All aircraft except the C-130 will provide airlift from the CONUS to the APOD.

12) Only the C-5B and C-17 are capable of carrying outsized cargo.

13) The C-17 is capable of airdropping all types of cargo, even outsized equipment.

14) The C-141 and C-17 are the only two aircraft involved in airdrop operations.

15) The twenty day period of deployment can be reflected in the linkage of four five-day periods.

16) The twenty day period is a reasonable time frame whereby airlift will have played its most critical role.

17) Attrition will be modeled as a percentage of total missions of a given mission and aircraft type.

18) The Battalion Headquarters, F-16 squadron, and Truck Company will not be delivered to the front.

19) All aircraft lost due to attrition are assumed to be carrying cargo.

20) The C-141 fleet is assumed to possess a fuel inert system due to its extreme vulnerability to a threat environment.

21) OSAN AB is assumed to be unusable due to runway damage.

22) All bases are assumed to be operational 24 hours/day.

Input Parameters

The number of sorties generated by the airlift force depends upon the number of aircraft available for the operation and the time in days required for an aircraft to transit the system and return to home station. The number of aircraft available for the airlift has been estimated to approximate the normal aircraft by type available to 22nd Air Force. It is assumed that 21st Air Force crews

and aircraft would augment 22nd Air Force by flying a percentage of the normal channel missions. This scenario assumes the following aircraft will participate in the contingency operation to Korea.

Table I

Aircraft Availability

Aircraft Type	Home Station	Organization	Number Available
C-5A/B	Travis AFB	60th MAW	40
C-17	west coast	?	0
C-141B	McChord AFB	62nd MAW	
C-141B	Travis AFB	60th MAW	110(total)
C 141B	Norton AFB	63rd MAW	
Cargo 747	CRAF		30
DC-8	CRAF		20
C-130	Clark AFB	374th TAW	16
C-130	Yokota AFB	316th TAG	16
C-130	McChord AFB	62nd MAW	16

For the purposes of the experimental design, the number of C-5B's was varied from 20 to 60 available for the operation. The assumption of 60 C-5's would require 21st Air Force participation in the airlift from Dover AFB. The use of 20 or 40 C-5's would assume the airlift for this particular model would come largely from Travis AFB, with the 22nd Air Force C-5 crews and aircraft picking up the routine channel missions which would still be required to be supported. The number of C-17's was varied from the existing zero level to the addition of 20 or 40 aircraft. The C-130's were varied from an average of 60 to a low of 40 and a high of 80. The 60 and 80

levels assume additional support from another C-130 wing stationed in the United States, but placed on rotation to the Pacific for a thirty day period.

The number of days required for an aircraft to complete an entire mission is dependent upon the aircraft speed and the distance between airfields. The speed of the aircraft used for computations is not the true airspeed (TAS) but rather the groundspeed, which is computed for each mission leg, depending upon forecast wind factors. The wind factors were obtained from HQ MAC/XOSS. Even though wind factors can change appreciably based upon the time of year, the average values obtained from MAC will suffice for the purposes of the model. The intertheater aircraft routing requires an initial departure from Travis, an enroute refueling stop at Elmendorf, and either a flight to the APOD (Kwangju) or direct to the FOL (Suwon). In both cases the crews will recover to Yokota Air Base for refueling and crew changes before the return trip back through Elmendorf to Travis AFB. Standard ground times and an expected maintenance delay time were added to the figures to determine an estimate of the round trip time required for each aircraft type. Intratheater times are calculated using the same criteria with the following results:

Table II

Aircraft Usage Rates(days)

Aircraft Type	Inter (i)	Intra (i)
C-5B	2.2	N/A
C-17	2.0	0.20
C-141	2.1	N/A
C-747	2.0	N/A
DC-8	2.0	N/A
C-130	N/A	0.25

Attrition for airlift aircraft is a subjective issue with little actual data to support a particular viewpoint, yet there is no disagreement that the loss of a substantial percentage of our airlift resources could prove disastrous to any major military operation. Strategic airlift resources are currently unable to meet desired goals in terms of metric ton-miles per day with no attrition, so even a low level of attrition could prove to be significant to the war effort. As pointed out in a recent article, "more than ever before, aircraft will be destroyed far faster than replacements can be built" (10:341).

The value of determining the expected attrition of airlift aircraft is not so much to model the various usage rates for each aircraft type, but rather to use aircraft attrition as an approximation of the average loss in combat power for each type of aircraft.

Aircraft attrition has been approximated by first assigning an attrition factor to each aircraft type based upon the mission to be flown. The four mission types were assigned a subjective ranking from zero to one in 0.25 increments based upon which mission could be expected to result in a higher level of attrition. The airdrop mission was assumed to possess the highest risk and was assigned a value of one, whereas a mission to the APOD was considered the safest and assigned a rating of 0.25. Missions departing a safe location and destined for another relatively safe location, such as the FOL, were assigned a value of 0.5.

These rating were purely subjective and in reality enemy forces could focus on one particular method of delivery and without near total air supremacy, cargo aircraft on any of these missions would be highly susceptible to enemy fire. Nevertheless, these values do provide a basis for useful comparison for the purposes of this model. Each aircraft type was also assigned a relative ranking based upon expected survivability in a threat environment. The baseline as far as the least survivable aircraft was established with the two

commercial (CRAF) aircraft, each receiving a value of one. Based upon such factors as combat aircrew training, aircraft size, redundancy of major systems, and fire suppressant capability (i.e. foam in wings, liquid nitrogen systems, etc.) the C-5B, C-130, and C-141 were each assigned a value of 0.75. An expanded discussion of the C-141's vulnerability due to fuel vapor in its tanks will follow in this chapter. The C-17 was assigned a rating of 0.5, making it the least vulnerable aircraft, due to proposed design enhancements over current aircraft in the airlift inventory. The two factors for mission type and aircraft type were combined to produce an overall attrition factor for each aircraft flying a particular mission. While somewhat simplistic, this approach appears more realistic than assuming a constant attrition rate regardless of aircraft or mission.

As stated in the list of assumptions, these attrition factors are based upon the C-141B possessing a fuel inert system. The type of system is not at issue, but while the C-5B has a liquid nitrogen system (LN2), and the C-130 has the fire suppressant foam in its wings, the C-141 has no such capability. On an intertheater airdrop mission, a typical C-141B would probably have fuel only in its main fuel tanks (approximately 44,000 lbs) by the time it approached the drop zone. That would mean the extended and aux range tanks would be filled with fuel vapor, waiting for a source of ignition. Discussions with Mr.

Redman (25) and Major Maroney (16), both from ASD/XRS, highlighted the importance of a fuel inert system in terms of the vulnerability of an aircraft. Their studies emphasized that a large percentage of the vulnerability of an aircraft is directly related to the inherent risk of operating an aircraft in a threat environment with fuel tanks filled with fuel vapor (ullage). The MAC aircraft most vulnerable to this ullage risk is also currently the "workhorse" of MAC, the C-141B Starlifter. If the C-141B is to be used in an environment where attrition is a factor, the installation of a fuel inert system on the C-141B should be given high priority by MAC (25). This model assumes a fuel inert system on the C-141B. The analysis chapter of this thesis discusses the impact in sorties and combat power delivered if the C-141B continued to have no such system.

The expected survivability of an aircraft can be analyzed by discussing survivability in terms of its two major components, vulnerability and susceptibility (19). Vulnerability refers to the characteristics of the aircraft, such as redundant flight control systems (routed through different locations of the fuselage), fire suppressant systems, aircrew combat training, etc. (19:5). Susceptibility refers to the threat exposure which relates more t- the characteristics of the mission (19:4). Threats for a certain mission type can be minimized by avoiding the threat, or minimizing exposure

to that threat. Minimizing exposure can be achieved by aircraft speed or perhaps more critically, aircraft altitude and terrain following capability. Vulnerability and susceptibility are combined in an effort to quantify survivability of an aircraft. This simplified aspect of the model combines the susceptibility attrition factor (mission type) with the vulnerability attrition factor (aircraft type) to determine an overall factor of attrition. When the scenario specifies a given attrition rate, (i.e. five per cent), each aircraft and mission combination is multiplied by its respective factor of attrition.

The number of aircraft available for a given period is determined by subtracting the summation of previous attrition values for that specific aircraft type. The summation of attrited aircraft is then used to estimate the loss of combat power due to the loss of aircraft. An average value of combat power per aircraft load was estimated by varying the number of aircraft in the model, relaxing some of the constraints to remain feasible, and examining the changes in the objective function. The combat power per load estimates and the estimates of the number of loads (aircraft) lost were then used to determine the combat power lost due to aircraft attrition. This attrition of combat power due to aircraft attrition was modeled by including the variable [ATT(i)], the expected attrition by aircraft type, in the objective

function. The coefficient for each aircraft attrition variable is the constant [CPLOST(i)], the estimated value of combat power for each aircraft type. This formulation allows the objective function to be decreased by the combat power lost on the aircraft which were attrited by the model. The attrition factors and estimated combat load per aircraft estimates are included in the following table:

Table III

Attrition and Combat Power/Load Factors

Aircraft	Mission	Attrition Factor	Estimated Combat Power/Load
C-5B	Inter/APOD	1.0	0.1075
C-17	Inter/APOD	0.75	0.131
"	Inter/Direct	1.0	"
"	Inter/Airdrop	1.5	"
"	Intratheater	1.0	"
C-141	Inter/APOD	1.0	0.061
"	Inter/Direct	1.25	"
"	Inter/Airdrop	1.75	"
C-747	Inter/APOD	1.25	0.143
DC-8	Inter/APOD	1.25	0.062
C-130	Intratheater	1.25	0.025

It may be noted that the estimated combat power per load is higher for the C-17 than it is for the C-5. This factor is due to the higher value CPI(1) given to aircraft loads that arrive directly to the front. If the C-5 could deliver it's cargo directly to the objective area, the combat power load factor for the C-5 would be closer to 0.15. This factor does not imply that all C-17 loads

would be delivered directly to the front, but the mean value for the cargo delivered would be higher for the C-17 than it would be for the C-5.

The utilization rate of the aircraft determines how many hours an aircraft can be expected to be airborne in a 24 hour period. Utilization (UTE) rate is calculated in peacetime by dividing the total programmed flying hours by the primary aircraft authorized (PAA), then dividing that figure by 360 to arrive at a daily UTE rate. The actual surge capability UTE rates are classified, but they can be expected to exceed current actual flying hours. Estimated UTE rates are found in Table IV. UTE rates are dependent upon aircraft type and type of mission. In developing the model, the maximum UTE rates varied as the number of aircraft were increased or decreased in the model.

Table IV

Aircraft Capabilities

	UTE RATES						
A/C	Intertheater APOD/Direct	Intra	Airdrop	Pax Capacity	TAS	Gnd Times (hours)	463L Pallets
C-5	11/0	0	0	320	430	3.25	36
C-17	14/13	12	10	160	430	2.0	18*
C-141	12.5/10	10	8	155	410	2.25	13**
C-747	10/0	0	0	0	430	3.6	36
DC-8	10/0	0	0	364	430	2.8	0
C-130	0/0	4	0	64	270	2.25	6

* approximately 10 for airdrop
** approximately 8 for airdrop

The ramp capacity of an airfield can play a critical role in determining the value of that airfield as an operating base. As determined in the previous chapter, ramp saturation occurs when all available parking spots are occupied. The expected ground time for each aircraft helps determine how many of that type aircraft are capable of parking on the airfield. HQ MAC/DEP has a computer data base consisting of hundreds of airfields which have been surveyed to determine the maximum aircraft on the ground (MOG) based upon aircraft and airfield characteristics. APOD ramp capacity for the model was varied by allowing the ramp to vary in capacity by 25 per cent from the actual ramp capacity at Kwangju (i.e. C-5 ramp capacity was varied from six to ten). Table V contains the ramp capacity by aircraft type.

Table V

Ramp Capacity

Airfield (AFB)	Location Identifier	C-5B	C-17	C-141B	C-747	DC-8	C-130	F-16
Travis	KSUU	42	97	67	60	60	N/A	N/A
Elmendorf	PAED	16	45	39	30	30	N/A	N/A
Yokota	RJTY	7	16	15	8	8	21	N/A
Kwangju	RKJJ	8	17	12	10	10	18	40
Suwon	RKSW	3	15	12	5	5	17	N/A

Another area affecting airfield capability is the ability to efficiently process the loading and unloading of equipment brought in by air. The availability of heavy equipment, such as K-loaders, increases the productivity of the ALCE personnel. For the purpose of this model, the APOD is assumed to be able to handle 1000 pallet equivalents per day, while the FOL can handle 200 pallet equivalents per day. Pallet equivalents are used as a common measure of cargo type since not all cargo is palletized. As the ALCE arrives at the APOD the mathematical formulation of the constraint allows the material handling capability of the airfield to increase. The model varied the MHE capacity of the APOD from 700 to 1300 pallets per day.

The units will arrive to the FOL by a combination of overland delivery from the APOD, intratheater airlift from the APOD, intertheater airlift from the United States, and intertheater airlift via airdrop missions. Direct delivery is provided by the C-141B and, if available, the C-17. Only cargo required at the FOL is allowed to be shipped via direct delivery. All unit types (from 1-Y) will be allowed to be shipped to the APOD and all aircraft types except the C-130 are involved in intertheater airlift.

Another factor varied in the model is the ground distance from the APOD to the FOL expressed in terms of days of travel (in increments of one period). While this

factor may not change significantly for any given scenario, the impact of the distance between the APOD and the FOL is important and should be examined. Distance in travel periods for the experimental design was allowed to vary from zero days between APOD and FOL to 10 days travel between the two fields.

The planned allowable cabin load (ACL) for an aircraft depends upon the type of cargo to be carried, cargo weight, estimated enroute flight time, forecast weather conditions, and the availability of air refueling. Air refueling would enhance the payload and sortie generation capability of the C-5, C-17, and C-141, but for the purposes of this model air refueling is assumed not to be available due to higher priority requirements for the SAC aircraft. The inclusion of air refueling for this scenario would require recomputing the enroute time from Travis AFB to Korea deleting the requirement to refuel at Elmendorf AFB, and adding additional cargo for a refueling contact point near Alaska. The allowable cabin load for the aircraft type was extracted from data obtained from the office of the director of Program Analysis and Evaluation, in Washington, D.C. (9). The cargo capability in tons is found in the following table, and is based upon a 3100 nautical mile critical leg.

Table VI

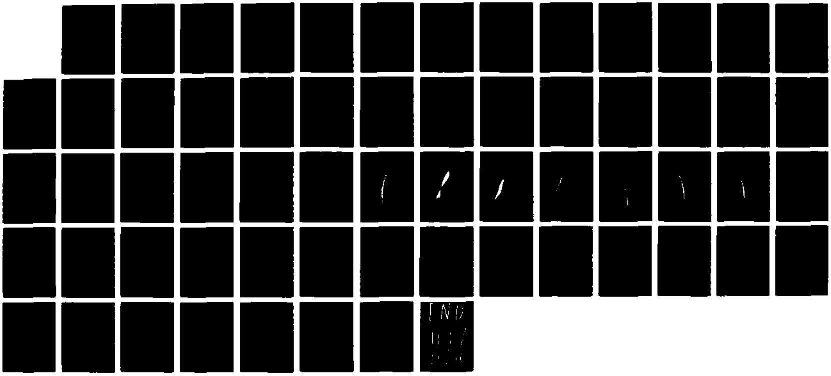
Cargo Capability (tons)

Aircraft Type	Outsize	Oversize	Bulk	Personnel (people)
C-5B	53	59	83	320
C-17	47	32	37	160
C-141	00	25	30	155
C-747	00	71	97	00
DC-8	00	00	41	364
C-130	00	00	13.8	64

Units required to be airlifted are dependent upon the scenario. This model assumed the same units as described in Cooke's thesis (7:83). The following data was extracted from FM 101-10-1 and Cooke's thesis:

AD-A185 269 THE USE OF MATHEMATICAL PROGRAMMING AND RESPOOSE SURFACE METHODOLOGY IN O..(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI.. R F HAILE 2/2

UUCLASSIFIED DDC 86 AFIT/GOR/OS/86D-4 F/G 15/5 NL



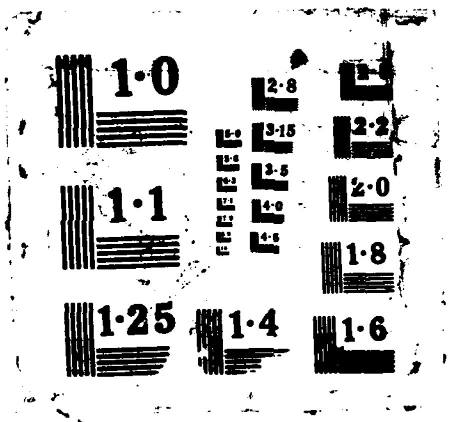

1·0
2·8
2·2
2·0
1·1
1·8
1·25
1·4
1·6

Table VII

Units To Deploy (tons)

Type	Number	Outsize	Oversize	Bulk	Pax	Daily Supply Reqd
Airborne Battalion	9	0	1400	268.7	1400	149.2
Airborne Battalion HQ	3	73	0	0	305	0
Air Assault Battalion	9	152	990.3	574.7	1960	446
Mechanized Battalion	9	2055	3885	151.7	2052	400.2
155mm Art. Battalion	3	139	1243	91.5	710	212.85
F-16 Sqdn *	3	0	249.6	155.4	472	102.5
Medium Truck Company **	3	1098	405	270.0	250	55.7
ALCE	2	176	2213	244.0	400	73.8

* 18 aircraft
** 180,000 ton-miles/day

In order to quantify the amount of combat power delivered to the front, the following inputs must be examined: the capability of the aircraft to deliver the cargo, the quantity of men and equipment per unit to be delivered, the values attached to the three MOE's of

combat power and the scaling developed for delivering combat power over time. The first two have been discussed, and the next two were developed from previous thesis efforts and the Army Command and General Staff College (7:83). The data is presented in the following tables:

Table VIII

Measures of Effectiveness

Type	Fire Power	Anti-Tank Strength	Defensive Frontage
Airborne Battalion	4	19.5	4
Airborne Batt. HQ	0	0	0
Air Assault Batt.	6	28.5	4
Mechanized Batt.	8	40	6
155 Art. Battalion	3	3	0
F-16 Sqdn	8	36	0
Medium Truck Company	0	0	0
ALCE	0	0	0

Table IX

Combat Value Over Time

Period	Combat Value Factor
1	2.5
2	1.8
3	1.3
4	1.1

Units destined for the front that are airdrop capable are included in the scenario, along with the only two aircraft capable of intertheater airdrop missions, the C-141B and the proposed C-17. The scenario allows the C-141 to airdrop supplies and personnel, while the C-17 is assumed to possess the capability to airdrop all four types of cargo. While not yet operational, the C-17 is being developed with the capability to airdrop outsized cargo, and that capability is included in the development of this scenario (2). The quantity of cargo delivered by airdrop operations is constrained by the total number of airdrop capable aircraft available, the number of pallets per aircraft in the airdrop configuration, and the rigging capability of the army riggers. This model assumes an army cargo rigging capability of 400 pallets per day, and it also assumes that all C-141B and C-17 are airdrop capable aircraft. Table IV lists the pertinent airdrop data.

Having determined the input parameters through a combination of historical data and a subjective estimation of the range of values for the scenario generated for this model, the following chapter will analyze the linear program and response surface outputs.

V. Analysis

Introduction

While linear programming remains a powerful and viable analytical tool, it does have limitations. The standard sensitivity analysis of ranging the right hand sides allows for only single parameter analysis. The restriction of varying only one parameter at a time would require an inordinate number of model runs to fully examine the sensitivity of the model to changing inputs. An alternative methodology, developed by Smith and Mellichamp (29), allows the use of response surface methodology and multivariate regression analysis to perform a multidimensional impact analysis of the model.

This methodology uses regression analysis as a curve fitting tool to estimate the coefficients of the mathematical equation used to describe the response surface. The generated response surface provides a means of depicting the different relationships between the factors of the model.

Experimental Design Development

The experimental design developed by Box and Behnken (3:461) was selected due to its rotatability and nearly orthogonal design. Even though the constant term β_0 and the quadratic estimates β_{ii} are correlated, the correlation is negligible (see Table X), and the overall design still possesses a high degree of orthogonality. The

Table X

CORRELATION OF ESTIMATES

CORRB	INTERCEP	X2	X3	X4
INTERCEP	1.0000	0.0000	0.0000	0.0000
X2	0.0000	1.0000	0.0000	0.0000
X3	0.0000	0.0000	1.0000	0.0000
X4	0.0000	0.0000	0.0000	1.0000
X5	0.0000	0.0000	0.0000	0.0000
X15	0.0000	0.0000	0.0000	0.0000
X22	-0.5855	0.0000	0.0000	0.0000
X24	0.0000	0.0000	0.0000	0.0000
X25	0.0000	0.0000	0.0000	0.0000
X34	0.0000	0.0000	0.0000	0.0000
X35	0.0000	0.0000	0.0000	0.0000
X44	-0.5855	0.0000	0.0000	0.0000
X55	-0.5855	0.0000	0.0000	0.0000

CORRB	X5	X15	X22	X24
INTERCEP	0.0000	0.0000	-0.5855	0.0000
X2	0.0000	0.0000	0.0000	0.0000
X3	0.0000	0.0000	0.0000	0.0000
X4	0.0000	0.0000	0.0000	0.0000
X5	1.0000	0.0000	0.0000	0.0000
X15	0.0000	1.0000	0.0000	0.0000
X22	0.0000	0.0000	1.0000	0.0000
X24	0.0000	0.0000	0.0000	1.0000
X25	0.0000	0.0000	0.0000	0.0000
X34	0.0000	0.0000	0.0000	0.0000
X35	0.0000	0.0000	0.0000	0.0000
X44	0.0000	0.0000	0.1786	0.0000
X55	0.0000	0.0000	0.1786	0.0000

CORRB	X25	X34	X35	X44
INTERCEP	0.0000	0.0000	0.0000	-0.5855
X2	0.0000	0.0000	0.0000	0.0000
X3	0.0000	0.0000	0.0000	0.0000
X4	0.0000	0.0000	0.0000	0.0000
X5	0.0000	0.0000	0.0000	0.0000
X15	0.0000	0.0000	0.0000	0.0000
X22	0.0000	0.0000	0.0000	0.1786
X24	0.0000	0.0000	0.0000	0.0000
X25	1.0000	0.0000	0.0000	0.0000
X34	0.0000	1.0000	0.0000	0.0000
X35	0.0000	0.0000	1.0000	0.0000
X44	0.0000	0.0000	0.0000	1.0000
X55	0.0000	0.0000	0.0000	0.1786

CORRB	X55
INTERCEP	-0.5855
X2	0.0000
X3	0.0000
X4	0.0000
X5	0.0000
X15	0.0000
X22	0.1786
X24	0.0000
X25	0.0000
X34	0.0000
X35	0.0000
X44	0.1786
X55	1.0000

three level rotatable design included seven variables requiring only 56 points plus an added point at the origin. This design provided efficient estimates of the β's for the second order polynomial.

The design was generated by selecting a range for the seven variables to be investigated, coding the variables, and running the linear program 57 times to generate the required objective function values. The notation for this particular design requires the lower limit of the individual variable to be coded as a "-1", the center point value of the variable to be coded as a "0", and the upper limit value to be coded with a "+1". The coded design is depicted in Appendix A. The seven variables selected to be investigated were: 1) the number of C-5B aircraft available, 2) the number of C-17 aircraft available, 3) APOD MHE capability, 4) aircraft attrition, 5) distance from the APOD to the FOL, 6) APOD ramp space, and 7) the number of C-130's available for the operation. The noncoded variable values are listed in Appendix B. The interaction and quadratic terms were generated using SAS. The SAS program is listed in Appendix C. The interaction terms, such as x12, refer to the interaction between two particular main effects. For this example, the interaction is between C-5's(x1) and C-17's(x2). In three dimensions, the plane generated by the x1 and x2 variables is twisted by the interaction term x12. The magnitude of the twist is determined by the coefficient for x12.(7:99)

Quadratic terms, such as x11, refer to the interaction of a main effect term with itself. The coefficient of a quadratic response expresses the degree of curvature of the response surface to that particular axis. For the full model, there are seven main effect terms, 21 interaction terms, seven quadratic terms, and the intercept term, for a total of 36 terms.

Response Surface Development

Utilizing a second order model with the appropriate interaction and quadratic terms, a response surface was generated. The linear programming model, consisting of 168 constraints and 288 variables, was run 57 times using the points of the design depicted in Appendix B. The seven variables were abbreviated using the following format:

Variables	Range
x1 = Number of C-5B aircraft	20 - 60 aircraft
x2 = Number of C-17 aircraft	0 - 40 aircraft
x3 = APOD MHE capability	700 - 1300 pallets/day
x4 = Aircraft Attrition	0 - 10 per cent
x5 = Distance APOD to FOL	0 - 2 periods
x6 = APOD ramp size capability	0.75 - 1.25 std ramp
x7 = Number of C-130 aircraft	40 - 80 aircraft

The interaction and quadratic terms are as specified in the SAS program in Appendix C.

Using Proc Stepwise with the "maxr" option, the SAS program brought each of the variables into the predicted equation. The program determined the best model based upon the number of variables allowed in the program. Since there is no variance in the output of a deterministic model, all the terms were initially included in the model. Proc Stepwise then determined the best one variable model, the best two variable model, and so on until all variables were included. A regression was run with all the variables in the model with the results listed in Figure 1. The prediction capability of the full model is shown in Table XI, which lists the linear programming output "y", the prediction value of the experimental design, and the percentage error for each of the 57 runs. From a combat value range of 113.05 to 297.63, the full model had a mean percentage error of 2.247 percent.

For comparison, a model with less than the total variables was analyzed. Using R^2, which measures the percentage of the variation of the dependent variable "y" which can be explained by the independent variables currently in the model, and the error sum of squares (SSE), a model with 12 variables was selected for comparison. Proc Stepwise, with the "maxr" option, selected the best 12 variable model. Proc Rsquare was also used to confirm that the correct variables were selected for the model. A regression of the 12 variable model produced the results shown in Figure 2. Table XII lists the dependent variable

SAS 1
16:52 TUESDAY, NOVEMBER 11, 1986

DEP VARIABLE: Y

COMBAT FIRE POWER
ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	35	111141.3	3175.465	35.492	0.0001
ERROR	21	1878.869	89.46996		
C TOTAL	56	113020.1			

ROOT MSE	9.458856	R-SQUARE	0.9834
DEP MEAN	196.6556	ADJ R-SQ	0.9557
C.V.	4.809859		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0
INTERCEP	1	194.185	9.458856	20.529
X1	1	0.7795417	1.930781	0.404
X2	1	22.25063	1.930781	11.524
X3	1	12.78904	1.930781	6.624
X4	1	-38.0216	1.930781	-19.692
X5	1	-43.9425	1.930781	-22.759
X6	1	1.234375	1.930781	0.639
X7	1	-0.00104167	1.930781	-0.001
X11	1	-3.06	3.861562	-0.792
X12	1	-0.69625	3.344211	-0.208
X13	1	-2.447	3.344211	-0.732
X14	1	1.1555	3.344211	0.346
X15	1	-5.95075	3.344211	-1.779
X16	1	-0.150875	3.344211	-0.045
X17	1	-0.003125	3.344211	-0.001
X22	1	-14.0293	3.861562	-3.633
X23	1	2.355625	3.344211	0.704
X24	1	-6.715	3.344211	-2.008
X25	1	12.361	3.344211	3.696
X26	1	-0.526875	3.344211	-0.158
X27	1	0	3.344211	0.000
X33	1	-3.20913	3.861562	-0.831
X34	1	-3.71075	3.344211	-1.110
X35	1	-8.7935	3.344211	-2.629
X36	1	1.569875	3.344211	0.469
X37	1	0	3.344211	0.000
X44	1	3.7145	3.861562	0.962
X45	1	0.036625	3.344211	0.011
X46	1	0.101875	3.344211	0.030
X47	1	0	3.344211	0.000
X55	1	22.21737	3.861562	5.753
X56	1	-0.071625	3.344211	-0.021
X57	1	0	3.344211	0.000
X66	1	-0.44325	3.861562	-0.115
X67	1	-0.003125	3.344211	-0.001
X77	1	0.677375	3.861562	0.175

Fig 1. Regression Output - Full Model

DEP VARIABLE: Y

COMBAT FIRE POWER
ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	12	110762.6	9230.217	179.900	0.0001
ERROR	44	2257.525	51.30739		
C TOTAL	56	113020.1			

ROOT MSE	7.162918	R-SQUARE	0.9800
DEP MEAN	196.6556	ADJ R-SQ	0.9746
C.V.	3.642367		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0
INTERCEP	1	189.9867	1.928193	98.531
X2	1	22.25063	1.462125	15.218
X3	1	12.78904	1.462125	8.747
X4	1	-38.0216	1.462125	-26.004
X5	1	-43.9425	1.462125	-30.054
X15	1	-5.95075	2.532474	-2.350
X22	1	-12.7173	1.97581	-6.436
X24	1	-6.715	2.532474	-2.652
X25	1	12.361	2.532474	4.881
X34	1	-3.71075	2.532474	-1.465
X35	1	-8.7935	2.532474	-3.472
X44	1	5.026457	1.97581	2.544
X55	1	23.52933	1.97581	11.909

Fig 2. Regression Output - Selected Model

Table XI

SAS 5
16:52 TUESDAY, NOVEMBER 11, 1986

OBS	ERROR1	Y	PRED
1	0.036797	144.126	139.011
2	0.038448	141.729	136.482
3	-0.044587	216.846	226.966
4	-0.040437	215.086	224.150
5	0.042202	223.841	214.777
6	0.047587	222.775	212.655
7	-0.017325	297.631	302.878
8	-0.017024	295.355	300.470
9	0.005771	194.330	193.215
10	0.005825	194.355	193.229
11	0.003192	191.664	191.054
12	0.003181	191.664	191.056
13	-0.005894	188.074	189.189
14	-0.005949	188.074	189.200
15	-0.003166	191.356	191.964
16	-0.003177	191.356	191.966
17	-0.044392	185.119	193.719
18	-0.044403	185.119	193.721
19	0.044555	268.329	256.884
20	0.044563	268.329	256.882
21	0.036278	245.704	237.102
22	0.036269	245.704	237.104
23	-0.091949	113.050	124.497
24	-0.091934	113.050	124.495
25	0.032557	164.758	159.563
26	-0.001675	246.312	246.725
27	-0.008920	188.490	190.187
28	0.037932	134.811	129.884
29	0.003316	125.035	124.622
30	-0.027407	184.351	189.546
31	-0.019796	243.943	248.870
32	0.010800	158.782	157.085
33	-0.000195	166.391	166.423
34	-0.000207	166.391	166.425
35	0.006686	251.561	249.890
36	0.006694	251.561	249.888
37	0.000159	216.923	216.889
38	0.000149	216.923	216.891
39	-0.011283	146.596	148.269
40	-0.011269	146.596	148.267
41	-0.048024	154.761	162.568
42	-0.004392	278.712	279.942
43	-0.040466	231.891	241.670
44	0.046065	166.817	159.471
45	0.007455	166.149	164.919
46	0.034960	231.123	223.316
47	-0.027069	264.029	271.375
48	0.055001	187.584	177.805
49	-0.003682	215.380	216.176
50	-0.005492	210.459	211.621
51	-0.013778	181.930	184.472
52	-0.015010	180.004	182.747
53	0.008071	145.173	144.011

Table XI(continued)

SAS 6
16:52 TUESDAY, NOVEMBER 11, 1986

OBS	ERROR1	Y	PRED
54	0.0055424	144.424	143.628
55	0.0169995	164.098	161.355
56	0.0151277	170.559	168.017
57	-0.0000000	194.185	194.185

SAS 7
16:52 TUESDAY, NOVEMBER 11, 1986

VARIABLE	LABEL	MEAN	MINIMUM VALUE	MAXIMUM VALUE
Y	COMBAT FIRE POWER	196.6555789	113.050000	297.631000
PRED	PREDICTED VALUE	196.6555789	124.495375	302.878458
ERR1		0.0224753	0.000000	0.091949

"y" from the linear program, the prediction value based upon the selected model and the percentage error for the 57 runs. Using only 12 of the 35 variables, the model produced a mean percentage error rate of only 2.687 percent, only 0.4 percentage error increase over the full 35 variable model (Figure 1). By comparison, using linear programming to examine the effects of the seven variables by varying the aircraft in increments of five, the APOD MHE in increments of 100 pallets/day, attrition by .5 percent, ramp space by 0.25 percent, and APOD to FOL distance by 1 day increments would require 1,843,200 separate runs of the linear programming model.

Verification

The 12 variable model includes the main effects of: 1) number of C-17's, 2) APOD MHE capability, 3) aircraft attrition, and 4) the number of days travel between the APOD and FOL. As expected, the β coefficients for the number of C-17's and the amount of MHE capability have positive signs indicating their positive effect on the objective function. Attrition and the distance from the APOD to the FOL have negative signs indicating their negative effect upon the delivery of combat power. As an additional check of the proposed model, the percentage error was plotted against the predicted value with the results listed in Figure 4 for the selected model and Figure 3 for the full model. The plots are quite good with only one or two points approaching the eight percent error

Table XII

SAS 12
16:53 TUESDAY, NOVEMBER 11, 1986

OBS	ERROR	X2	X3	X4	X5	X15	X22	X24	X25	X34	X35	X44	X55	Y	PRE
1	0.055262	0	0	1	1	0	0	0	0	0	0	1	1	144.126	136.578
2	0.037711	0	0	1	1	0	0	0	0	0	0	1	1	141.729	136.578
3	-0.033936	0	0	1	-1	0	0	0	0	0	0	1	1	216.846	224.463
4	-0.041777	0	0	1	-1	0	0	0	0	0	0	1	1	215.086	224.463
5	0.052767	0	0	-1	1	0	0	0	0	0	0	1	1	223.841	212.622
6	0.047753	0	0	-1	1	0	0	0	0	0	0	1	1	222.775	212.622
7	-0.009569	0	0	-1	-1	0	0	0	0	0	0	1	1	297.631	300.507
8	-0.017143	0	0	-1	-1	0	0	0	0	0	0	1	1	295.355	300.507
9	0.022861	0	0	0	0	0	0	0	0	0	0	0	0	194.330	189.987
10	0.022992	0	0	0	0	0	0	0	0	0	0	0	0	194.355	189.987
11	0.008828	0	0	0	0	0	0	0	0	0	0	0	0	191.664	189.987
12	0.008828	0	0	0	0	0	0	0	0	0	0	0	0	191.664	189.987
13	-0.010068	0	0	0	0	0	0	0	0	0	0	0	0	188.074	189.987
14	-0.010068	0	0	0	0	0	0	0	0	0	0	0	0	188.074	189.987
15	0.007207	0	0	0	0	0	0	0	0	0	0	0	0	191.356	189.987
16	0.007207	0	0	0	0	0	0	0	0	0	0	0	0	191.356	189.987
17	-0.033159	1	0	0	1	0	1	0	1	0	0	0	1	185.119	191.468
18	-0.033159	1	0	0	1	0	1	0	1	0	0	0	1	185.119	191.468
19	0.053796	1	0	0	-1	0	1	0	-1	0	0	0	1	268.329	254.631
20	0.053796	1	0	0	-1	0	1	0	-1	0	0	0	1	268.329	254.631
21	0.046210	-1	0	0	-1	0	1	0	1	0	0	0	1	245.704	234.852
22	0.046210	-1	0	0	-1	0	1	0	1	0	0	0	1	245.704	234.852
23	-0.075215	-1	0	0	1	0	1	0	-1	0	0	0	1	113.050	122.245
24	-0.075215	-1	0	0	1	0	1	0	-1	0	0	0	1	113.050	122.245
25	0.030962	1	0	1	0	0	1	1	0	0	0	1	0	164.758	159.810
26	-0.011919	1	0	-1	0	0	1	-1	0	0	0	1	0	246.312	249.283
27	-0.014956	-1	0	-1	0	0	1	1	0	0	0	1	0	188.490	191.352
28	0.047168	-1	0	1	0	0	1	-1	0	0	0	1	0	134.811	128.739
29	-0.028769	-1	0	1	0	0	1	-1	0	0	0	1	0	125.035	128.739
30	-0.036587	-1	0	-1	0	0	1	1	0	0	0	1	0	184.351	191.352
31	-0.021422	1	0	-1	0	0	1	-1	0	0	0	1	0	243.943	249.283
32	-0.006432	1	0	1	0	0	1	1	0	0	0	1	0	158.782	159.810
33	0.001934	0	1	1	0	0	0	0	0	1	0	1	0	166.391	166.070
34	0.001934	0	1	1	0	0	0	0	0	1	0	1	0	166.391	166.070
35	0.008121	0	1	-1	0	0	0	0	0	-1	0	1	0	251.561	249.535
36	0.008121	0	1	-1	0	0	0	0	0	-1	0	1	0	251.561	249.535
37	0.001792	0	-1	-1	0	0	0	0	0	1	0	1	0	216.923	216.535
38	0.001792	0	-1	-1	0	0	0	0	0	1	0	1	0	216.923	216.535
39	-0.008906	0	-1	1	0	0	0	0	0	-1	0	1	0	146.596	147.913
40	-0.008906	0	-1	1	0	0	0	0	0	-1	0	1	0	146.596	147.913
41	-0.076706	0	1	0	1	1	0	0	0	0	1	0	1	154.761	167.618
42	-0.022035	0	1	0	-1	-1	0	0	0	0	-1	0	1	278.712	284.992
43	-0.041086	0	-1	0	-1	-1	0	0	0	0	1	0	1	231.891	241.827
44	0.045040	0	-1	0	1	1	0	0	0	0	-1	0	1	166.817	159.627
45	-0.031364	0	-1	0	1	-1	0	0	0	0	-1	0	1	166.149	171.529
46	0.005209	0	-1	0	-1	1	0	0	0	0	1	0	1	231.123	229.925
47	-0.033181	0	1	0	-1	1	0	0	0	0	-1	0	1	264.029	273.090
48	0.044920	0	1	0	1	-1	0	0	0	0	1	0	1	187.584	179.520
49	0.014464	1	1	0	0	0	1	0	0	0	0	0	0	215.380	212.309

Table XII (continued)

OBS	ERROR	X2	X3	X4	X5	X15	X22	X24	X25	X34	X35	X44	X55	Y	PRE
50	-0.008714	1	1	0	0	0	1	0	0	0	0	0	0	210.459	212.309
51	-0.025711	1	-1	0	0	0	1	0	0	0	0	0	0	181.930	186.731
52	-0.036025	1	-1	0	0	0	1	0	0	0	0	0	0	180.004	186.731
53	0.020693	-1	-1	0	0	0	1	0	0	0	0	0	0	145.173	142.230
54	0.015427	-1	-1	0	0	0	1	0	0	0	0	0	0	144.424	142.230
55	-0.022108	-1	1	0	0	0	1	0	0	0	0	0	0	164.098	167.808
56	0.016395	-1	1	0	0	0	1	0	0	0	0	0	0	170.559	167.808
57	0.022098	0	0	0	0	0	0	0	0	0	0	0	0	194.185	189.987

VARIABLE	LABEL	MEAN	MINIMUM VALUE	MAXIMUM VALUE
X2	C-17 AIRCRAFT	0.000000	-1.000000	1.000000
X3	APOD MHE	0.000000	-1.000000	1.000000
X4	AIRCRAFT ATTRITION	0.000000	-1.000000	1.000000
X5	DIST TO FOL	0.000000	-1.000000	1.000000
X15		0.000000	-1.000000	1.000000
X22		0.421053	0.000000	1.000000
X24		0.000000	-1.000000	1.000000
X25		0.000000	-1.000000	1.000000
X34		0.000000	-1.000000	1.000000
X35		0.000000	-1.000000	1.000000
X44		0.421053	0.000000	1.000000
X55		0.421053	0.000000	1.000000
Y	COMBAT FIRE POWER	196.655579	113.050000	297.631000
PRE	PREDICTED VALUE	196.655579	122.244694	300.506611
ERR		0.026871	0.001792	0.076706

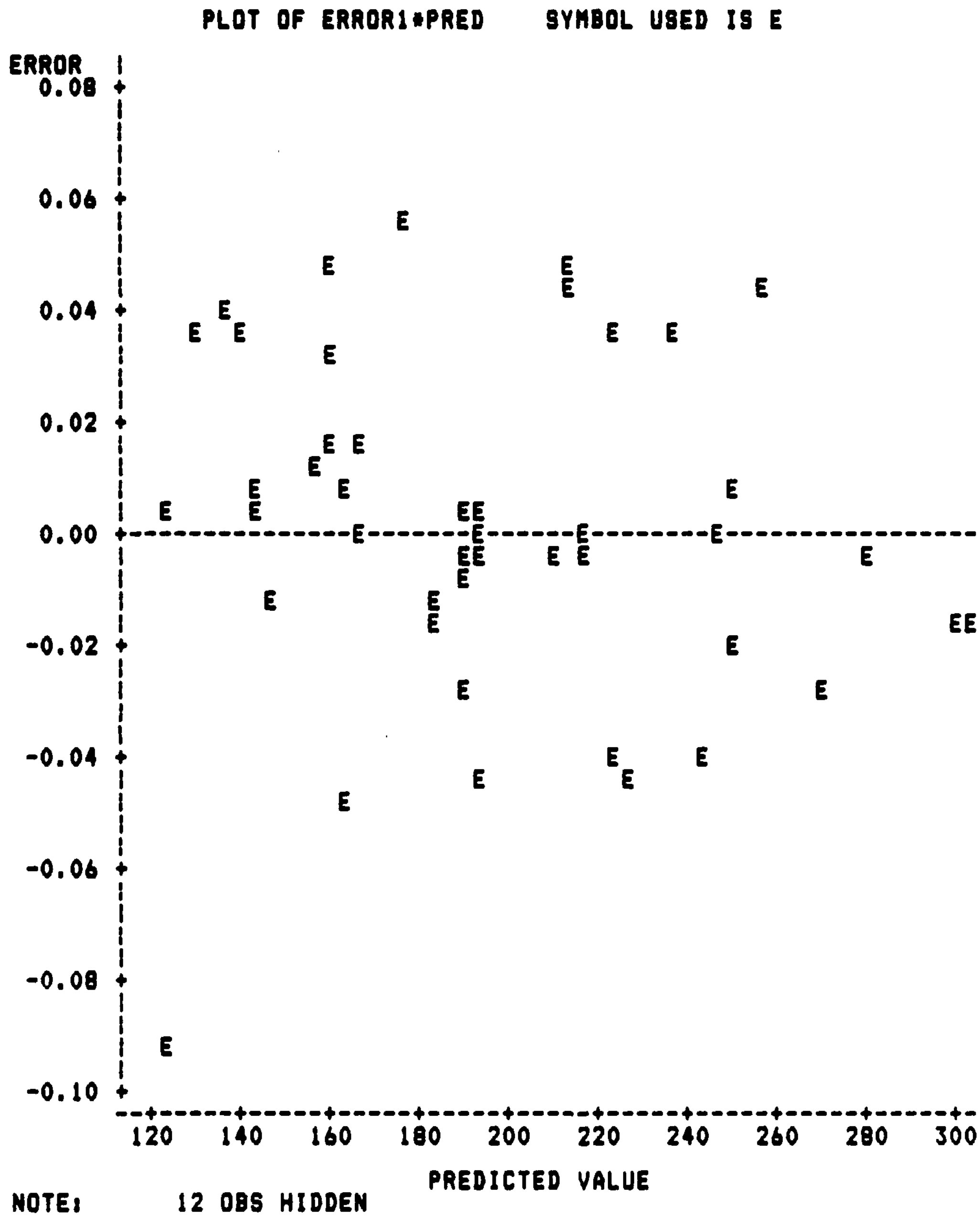


Fig 3. Percentage Error vs Predicted Value-Full Model

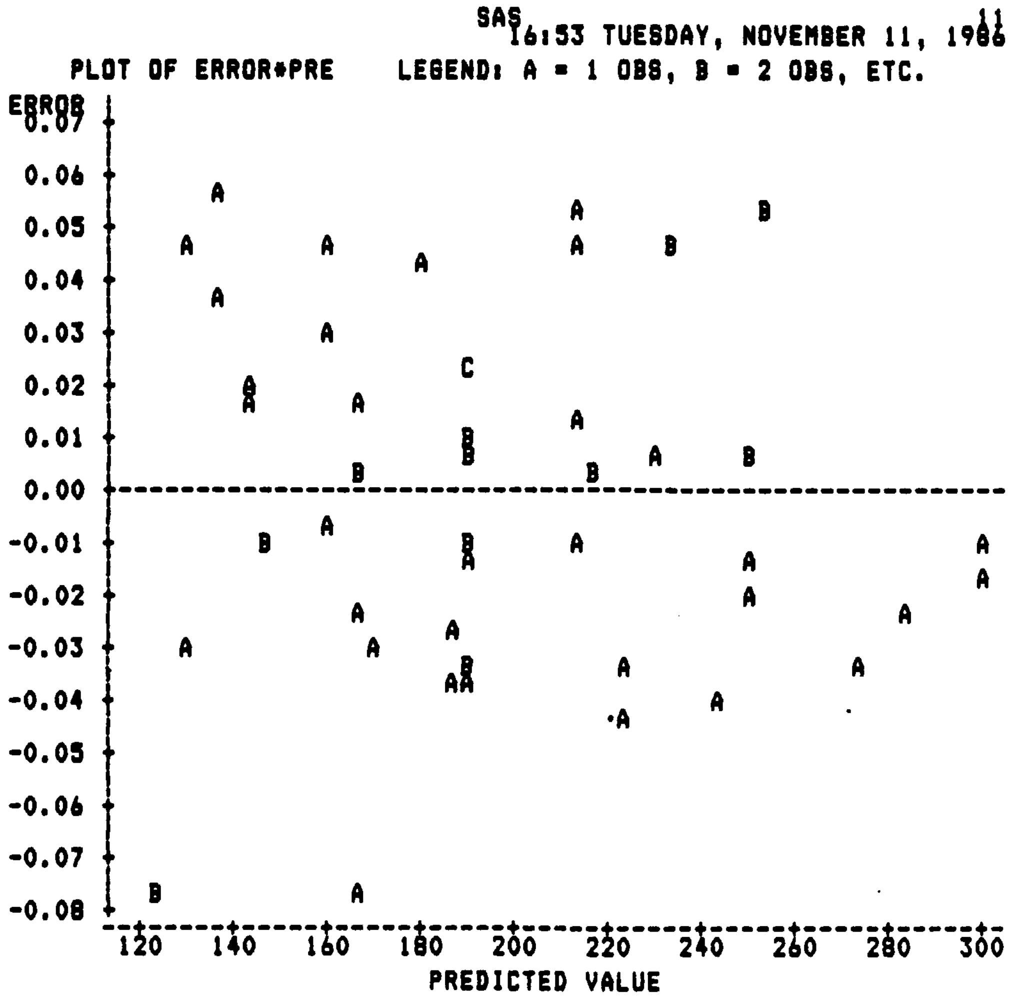


Fig 4. Percentage Error vs Predicted Value- Selected Model

rate for the selected model. The combination of 40 C-5's, 0 C-17's, 1000 pallets/day for MHE, five percent attrition, and 10 days travel between APOD and FOL, produces a response surface output approximately eight percent higher than the output of the linear programming model. Overall, the percentage error rate is approximately 2.68 percent, which means the response surface is quite accurate for prediction purposes. Approximately 98 percent of the variability in the objective function value can be explained by the 12 terms currently in the selected model. In addition, to the predicted outputs generated by the 57 runs of the LP model, ten mid-range and extreme points were generated for verification purposes. Even with these points, the mean percentage error rate was only 3.68 percent. The ten runs, model output, and predicted output, are shown in Table XIII.

Table XIII
Mid-range and Extreme Point Data *

C-5's	C-17's	APOD MHE	Attrition	Dist	APOD Ramp	C-130's	y	pred
30	30	1100	0.05	1	1.00	60	201.14	202.19
50	30	1000	0.05	2	1.25	40	183.42	195.60
60	5	1300	0.10	0	1.25	80	227.56	220.90
25	40	800	0.00	0	0.75	40	285.83	275.63
35	15	900	0.05	1	1.10	80	185.34	179.37
35	35	1250	0.00	1	0.75	40	252.26	260.84
50	20	700	0.10	1	1.00	50	147.37	147.91
45	35	1000	0.10	2	0.75	60	148.59	162.25
20	25	1200	0.00	1	1.00	80	244.41	250.48
30	10	1200	0.05	1	1.25	40	186.16	175.68

* Average percentage error = 3.68 percent

With an average error rate of only 3.68 percent for the mid-range and extreme point values and a 2.68 percent error rate for the 57 model runs, this response surface captures over 98 percent of the variability in the model and verifies that this quadratic response surface is an accurate indicator of the linear programming model responses. With over a 96 percent level of accuracy over a model with more than 1,843,200 possible points, the advantages of using the response surface equation to approximate the deterministic model outputs are obvious.

While the C-5 variables were included in the regression model, only the C-5 interaction with APOD to FOL distance was selected as a significant C-5 variable in determining the value of the objective function. This is not meant to imply that the C-5 is not a significant asset. Within this model, the number of C-5 aircraft available ranges between 20 and 60, and this availability of C-5's is sufficient to preclude it from becoming a major factor is estimating the value of the objective function.

Validation

Validation of the linear program and the response surface is a more difficult issue. There has been no real world event similar to the scenario generated for this thesis. The C-17 is not yet operational so its impact upon a given scenario can only be hypothesized. Certain aspects of the scenario parallel results of previous contingencies. The distance between the APOD and FOL and

any aircraft attrition adversely affects the timely delivery of combat power. In over 90 percent of the 57 model runs, MHE capacity at the APOD was a binding constraint, which reflects the importance of adequate MHE. This reality was highlighted in the 1981 Air Force MHE Conference, which stated that MAC was significantly short of its required MHE capability required to support a wartime scenario.(17:62) Throughout most of the various ranges of the model, the C-141 sorties and usage rates were shown to be binding constraints. Due to its flexibility, range, and cargo capacity, the C-141 has been designated as the "workhorse" of MAC, and the model validates that point of view. The C-141 was not selected as a critical variable required to explain the variance in the model because of the range of aircraft expected to be available for future operations. Regardless of whether the aircraft are operated by active duty, reserve, or Air Guard crews, the C-141 will remain in the airlift inventory for many years to come. This does not mean the C-141 is not important, only that it is expected to be available in sufficient numbers that it's availability has been determined not to be an issue for this particular model.

A general comparison can be made with Cooke's thesis, but so many variables have changed that a direct comparison would be inaccurate. Factors such as aircraft availability, constraint formulations, UTE rates, the reformulation of the excess supply variables, the inclusion

of aircraft attrition, the inclusion of combat power degradation due to aircraft attrition, different allowable cabin loads (ACL's), and a heavier emphasis on goal programming by Cooke make a direct comparison unrealistic. In general, both efforts provide similar insight into the importance of MHE and the probable impact of the C-17 on the timely delivery of combat power to the theater commanders. Additionally, this thesis highlights the critical impact of distance between the APOD and the FOL, and the affect of aircraft attrition on the timely delivery of combat power. Several other areas of interest were examined by the model with the following results.

Results

The results of this thesis will be presented in three separate but highly related methods. The first method examines the data using a one dimensional type of analysis, such as the standard right hand side ranging for linear programming. The second method uses a two-dimensional analysis by comparing and plotting combat power as a function of two of the independent variables. The third analysis integrates the first two methods by allowing the reader the opportunity to see some of the interrelated variables in terms of three-dimensional pictures.

One Dimensional Analysis. APOD MHE and to a lesser extent FOL MHE were binding constraints on a majority of the 57 program runs, highlighting the importance of adequate materials handling equipment in meeting the needs of the theater commanders.

Another important factor, which will be expanded in the two-dimensional and three-dimensional analysis, was the critical impact the distance from the APOD to the FOL had upon the value of the objective function. This point was especially true if the scenario did not include any C-17 aircraft in the airlift. Combat power decreased 54 percent in one comparison when the distance increased from zero to 10 days travel time. With 40 C-17's included, the decrease was reduced to 25 percent. While still a significant figure, the findings highlight the negative impact of distance and the positive impact of C-17 availability on the delivery of combat power.

The number of C-130's available was generally not a binding constraint in any of the generated scenarios. This point was due in part by the limited ability of the C-130 to airlift a large percentage of the cargo in this particular scenario. Over 83 percent of the cargo by weight consisted of either oversized or outsized cargo, neither of which can be loaded on the C-130.

The C-141 was utilized heavily in all the generated scenarios. Due to its intertheater and intratheater capabilities, the C-141 usage rates and maximum sorties were almost always binding constraints.

The impact of no fuel inerting system on the C-141 was examined from a users standpoint. The linear program had the advantage of knowing what the attrition rates would be for a specific scenario and the model would adjust C-141 usage accordingly. For a five percent scenario attrition rate, the model would use 10 percent less C-141's due to the expected loss rates. With a 10 percent attrition rate, there would be a 44 percent drop in the C-141 sorties as the model attempted to minimize the loss of combat power due to aircraft attrition. In reality attrition is not a known factor, and its impact upon the airlift force is not determined until the aircraft do or do not return from their assigned mission. Nevertheless, as a point of reference, if the current aircraft were allowed to fly each of the four types of missions every day of the 20 day operation, there would be 100 percent attrition of the C-141's before the end of the operation, an unacceptable loss in terms of men, aircraft, and cargo. A C-141 flying an airdrop mission already has the highest combined attrition factor, so if the C-141 is to be used in a role other than as an intertheater airlifter from CONUS to APOD, a fuel inerting system should become a high priority for MAC.

Two-dimensional Analysis. Plotting combat power as a function of the number of C-17's and the distance between the APOD and the FOL in periods (one period=5 days), revealed the importance of the C-17 as the distance

between the two bases increases. Setting the MHE capability at 1000 pallet equivalents per day and aircraft attrition at 5 percent, figure five shows how increasing the number of C-17's increases combat power delivered. While zero days travel between the APOD and FOL produces the greatest overall combat power delivered, the marginal advantage of each additional C-17 is highest when the distance is the greatest, as evidenced by the slope of each line. With a distance of zero, the marginal utility of additional aircraft is less, and eventually a saturation point is reached where other factors such as MHE or ramp space cause a decrease in combat power delivered. Combat power is a function of time; so if aircraft loads are delayed due to ramp or MHE saturation the combat value of that load will be reduced. For this particular scenario, with zero distance, the saturation point appeared to be between 30 and 35 aircraft. This saturation point would occur with the other two distances also, but at a higher level of total aircraft. The major point of this chart is to emphasize the increasing utility of the C-17 as the intratheater distances increase.

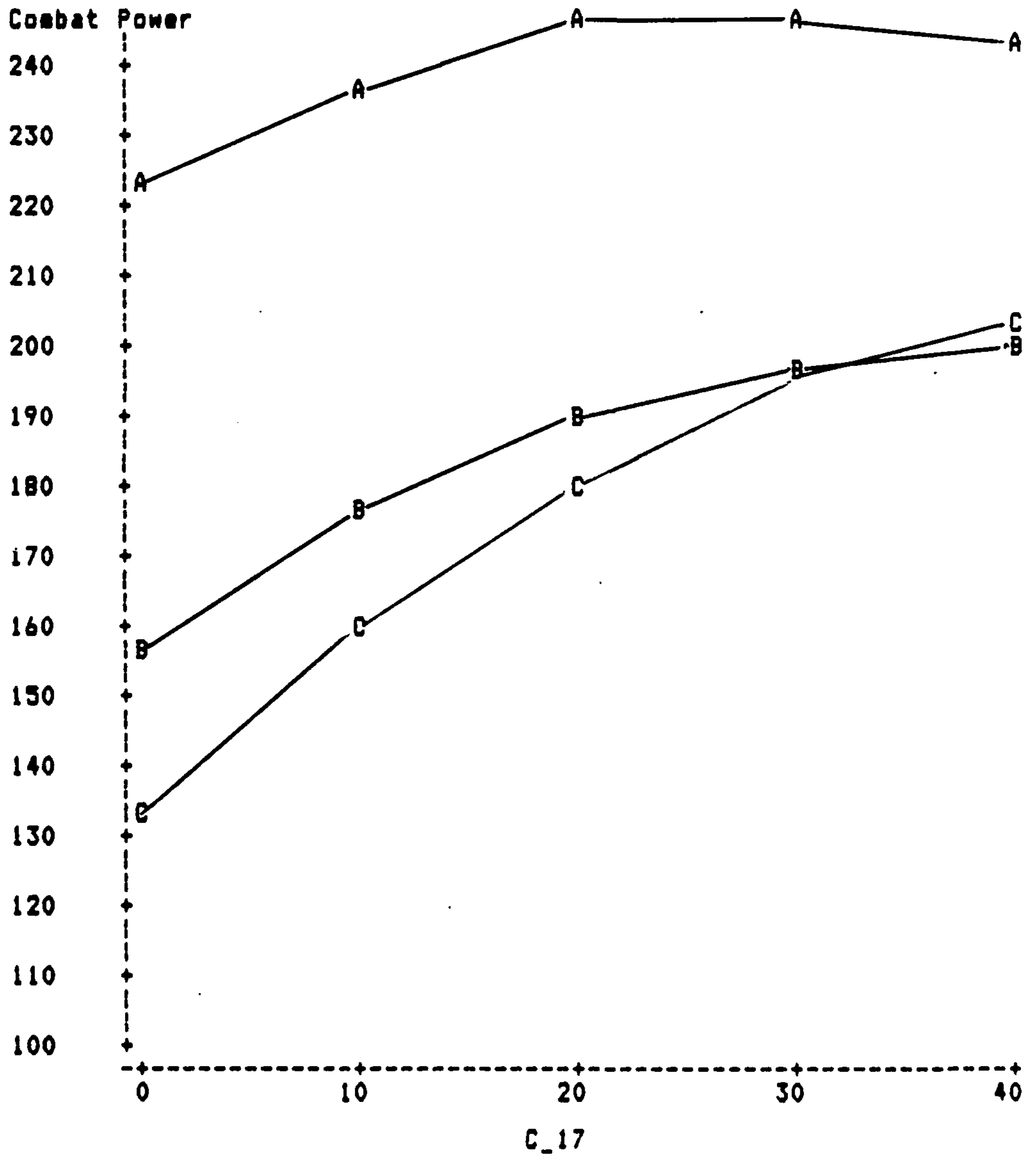
The plot of combat power as a function of MHE and the number of C-17's is depicted in figure six. As expected, as the amount of MHE and C-17's are increased, the amount of combat power increased. The marginal utility of each additional C-17 increased at a decreasing rate (decreasing slope) until eventually MHE capability is saturated and

SAS 1
13:34 WEDNESDAY, NOVEMBER 12, 1986

OBS	C_17	DIST_0	DIST_1	DIST_2
1	0	222.95	155.02	134.15
2	10	237.43	175.68	160.99
3	20	245.56	189.98	181.47
4	30	247.32	197.93	195.60
5	40	242.73	199.52	203.37

Combat Value - C17 vs Distance to APOD 2

PLOT OF DIST_0*C_17 SYMBOL USED IS A
PLOT OF DIST_1*C_17 SYMBOL USED IS B
PLOT OF DIST_2*C_17 SYMBOL USED IS C



NOTE: 1 OBS HIDDEN

Fig 5. Effects of C-17 Availability and Distance On Combat Power

additional aircraft provide no increased utility. The tradeoff between MHE and increased combat capability appears linear throughout the range of this graph. An increase in MHE capability of 300 pallets per day results in an incremental increase in combat power of 12 to 13 units, or approximately a 4.4 percent increase in terms of the maximum amount of combat power delivered for any one scenario.

A similar plot of combat power as a function of the number of C-17's and aircraft attrition appears in figure seven. As expected, the greater the attrition level, the less the incremental advantage of each additional C-17. The zero attrition level produced the largest overall values of combat power delivered as well as the largest marginal improvement of combat power per aircraft.

As noted earlier, the C-5 variable was not selected as a main effect in determining the value of the objective function. This finding indicates that increasing the number of C-5's from 20 to 60 will have no major effect upon the total value of combat power delivered. Using proc stepwise with the "maxr" option a regression equation with the C-5 variable as a factor in the model was selected to produce a regression equation where C-5's could be compared to distance (Appendix D). Using a response surface equation with 19 variables, a plot of combat power as a function of the number of C-5's available and the distance from the APOD to FOL (figure 8) verifies the earlier observation

OBS	C_17	MHE_700	MHE_1000	MHE_1300
1	0	142.23	155.02	167.81
2	10	162.89	175.68	188.47
3	20	177.20	189.99	202.78
4	30	185.14	197.93	210.72
5	40	186.73	199.52	212.31

Combat Value - C17 vs MHE Capability at APOD 4

PLOT OF MHE_700*C_17 SYMBOL USED IS A
PLOT OF MHE_1000*C_17 SYMBOL USED IS B
PLOT OF MHE_1300*C_17 SYMBOL USED IS C

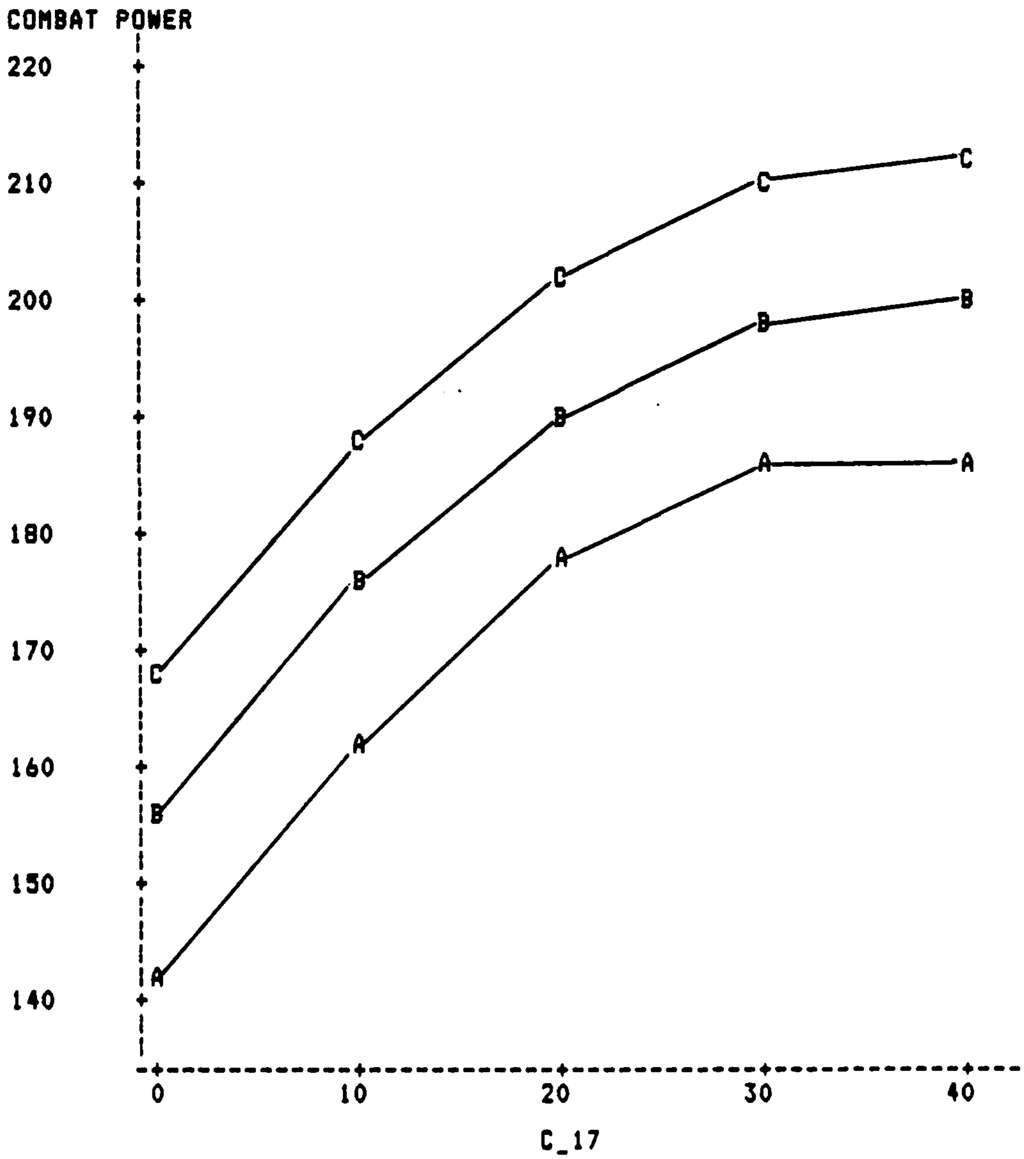


Fig 6. Effects of C-17 Availability and MHE On Combat Power

SAS 1
15:53 WEDNESDAY, NOVEMBER 12, 1986

OBS	C_17	ATTRIT_0	ATTRIT_5	ATT_10
1	0	191.35	155.02	128.74
2	10	215.37	175.68	146.04
3	20	233.04	189.99	156.99
4	30	244.34	197.93	161.58
5	40	249.28	199.52	159.81

Combat Value - C17 vs Attrition 2
15:53 WEDNESDAY, NOVEMBER 12, 1986

PLOT OF ATTRIT_0*C_17 SYMBOL USED IS A
PLOT OF ATTRIT_5*C_17 SYMBOL USED IS B
PLOT OF ATT_10*C_17 SYMBOL USED IS C

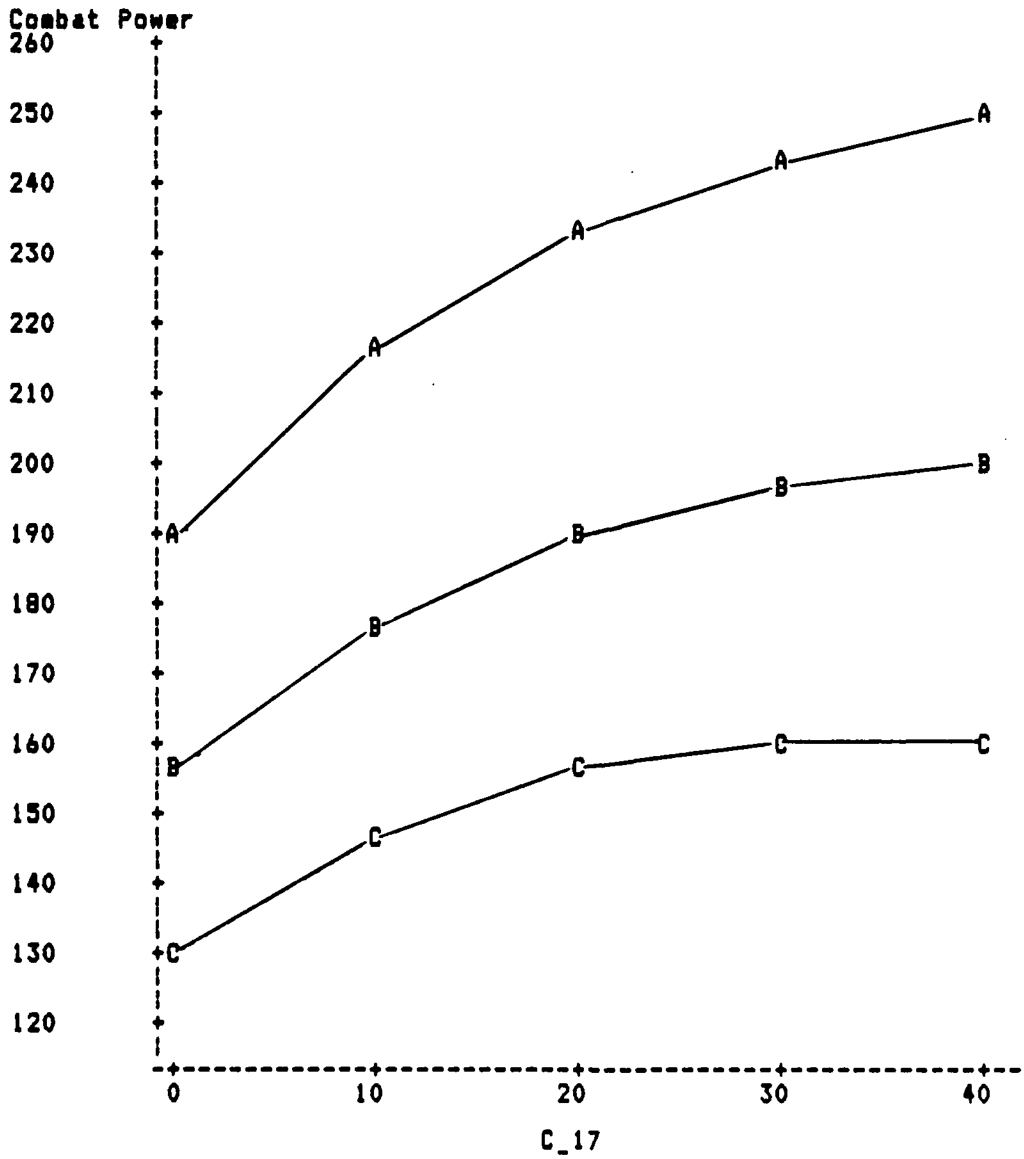


Fig 7. Effects of C-17 Availability and Attrition On Combat Power

that increasing the number of C-5's beyond the 20 in the model produces negligible improvement. While the current fleet of aircraft, including the 20 C-5's, do deliver a significant amount of cargo, the inclusion of up to 60 C-5's had a marginally positive impact on combat power if the distance from APOD to FOL is short. When compared with the C-17 versus distance graph (Fig. 5), it can be shown that while the C-5 delivered more cargo, the value of the C-5 decreased as the distance between the bases increased, while the value of the C-17 continued to increase.

Three-Dimensional Analysis. Several three-dimensional plots were produced using SAS/Graph, a Tektronics 4054, and an HP7220 plotter. In the first two cases combat power was examined while all but two of the main effects in the model were set to their mid-range value for the experimental design. The results highlight several key points discussed in the previous analysis. In order to visualize the impact of the independent variables, several pictures of the same plot were made from different angles. Figures 9, 10, 11, and 12 depict the effects of aircraft attrition and the number of C-17's on the amount of combat power delivered. The sequence of pictures rotate the plot approximately 45 degrees and increased the tilt approximately 40 degrees. The maximum point in the plane was the intersection where C-17's were at their maximum and attrition was at a zero level. The large decrease in combat power delivered can be

OBS	C_5	DIST_0	DIST_1	DIST_2
1	20	226.65	154.17	125.95
2	25	229.71	155.75	126.03
3	30	232.38	156.93	125.73
4	35	234.65	157.71	125.02
5	40	236.53	158.10	123.93
6	45	238.01	158.10	122.44
7	50	239.11	157.71	120.55
8	55	239.81	156.91	118.27
9	60	240.11	155.73	115.60

Combat Value - C5 vs Distance From APOD to FOL   2

PLOT OF DIST_0*C_5   SYMBOL USED IS A
PLOT OF DIST_1*C_5   SYMBOL USED IS B
PLOT OF DIST_2*C_5   SYMBOL USED IS C

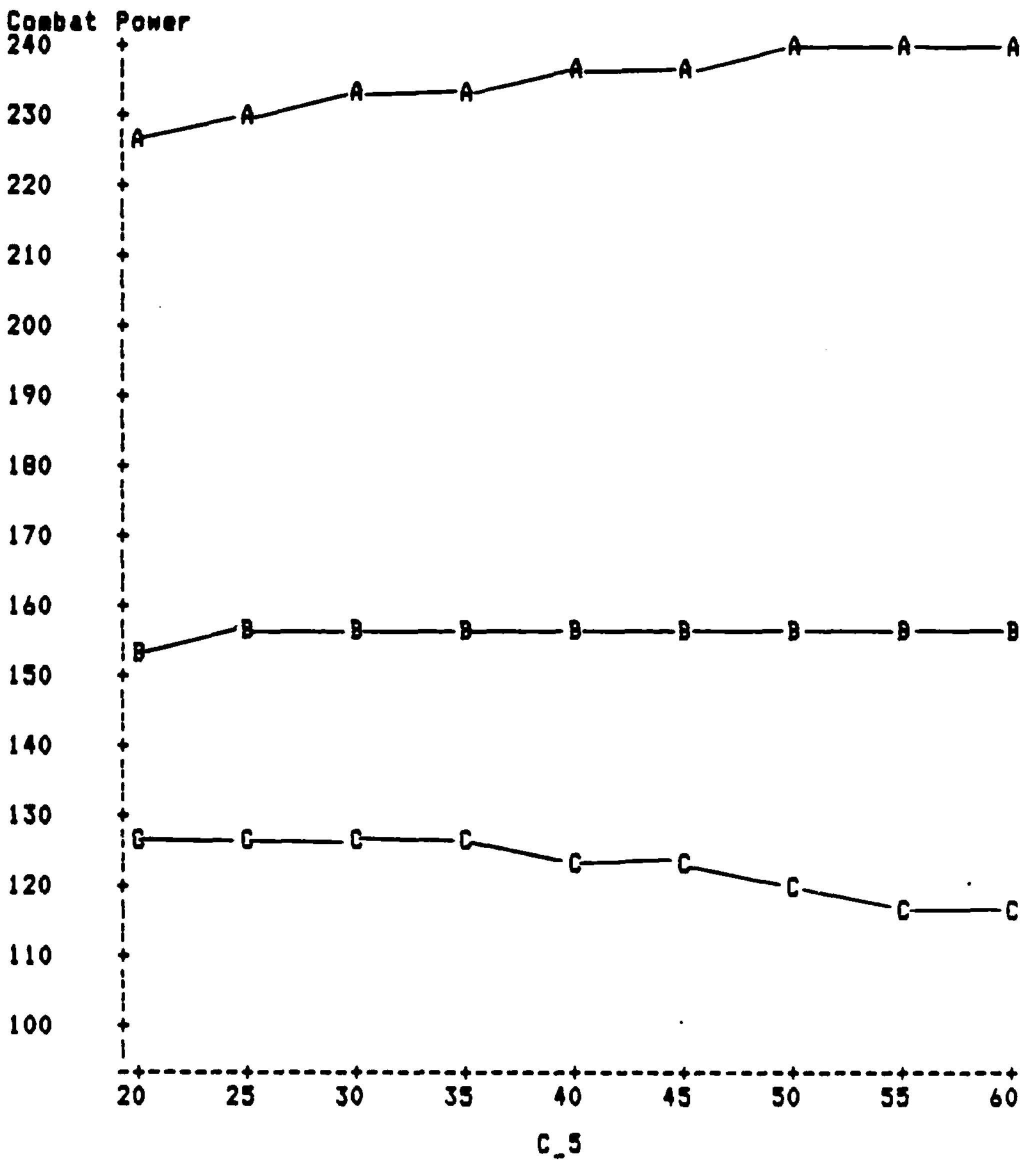


Figure 8. Effects of C-5 Availability and Distance On Combat Power

visualized by observing the maximum point (237.4), versus the point where attrition is 10 percent and there are no C-17's involved in the operation (116). The decrease represented a 51 percent reduction in combat power delivered due to the combination of high attrition and no C-17 aircraft in the scenario.

The second set of three-dimensional plots was used to analyze the effects of distance and attrition on combat power delivered. Again setting the other main effects at their mid-range values, the plane is depicted in figures 13, 14, 15, 16, and 17. The sequence rotated the plot 45 degrees and left the tilt set at approximately 70 degrees. The high point of the plane lies where distance equals zero and attrition equals zero. As distance and attrition increased, the plane slopes downward and towards its low point where attrition and distance are at the greatest value.

The third set of three-dimensional plots, figures 18, 19, and 20, depict the effect of attrition and the APOD to FOL distance upon the number of C-17's required to reach a certain combat power level. The combat power level was set at a relatively low value of 145 to illustrate the incremental increase in requirements for the C-17 as attrition and distance were both increased to ten percent and 10 days travel time respectively. With the other aircraft set at their mid-range values of 40 C-5's and 60 C-130's, the current force was initially capable of meeting

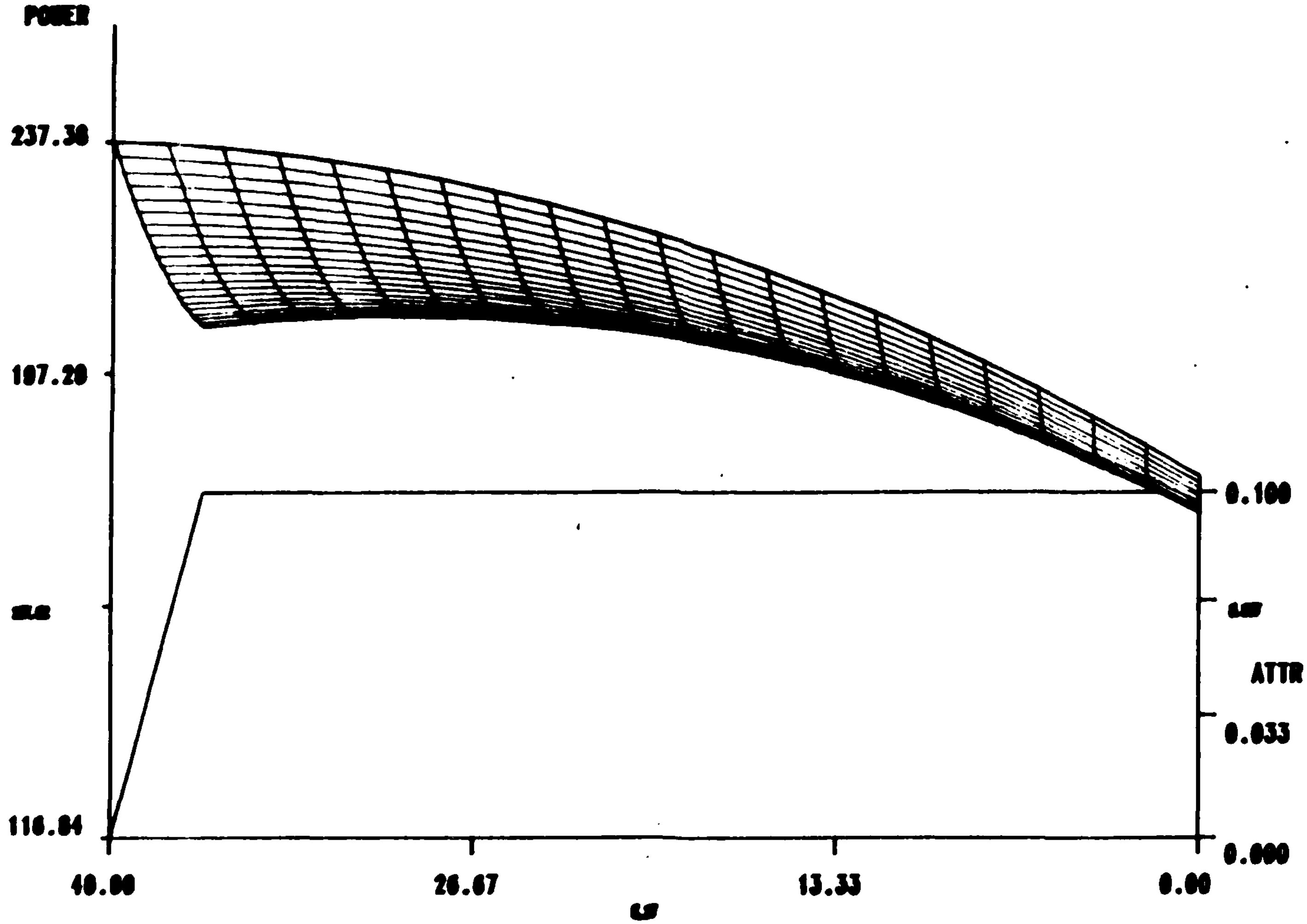


Fig. 9. Combat Power Delivered

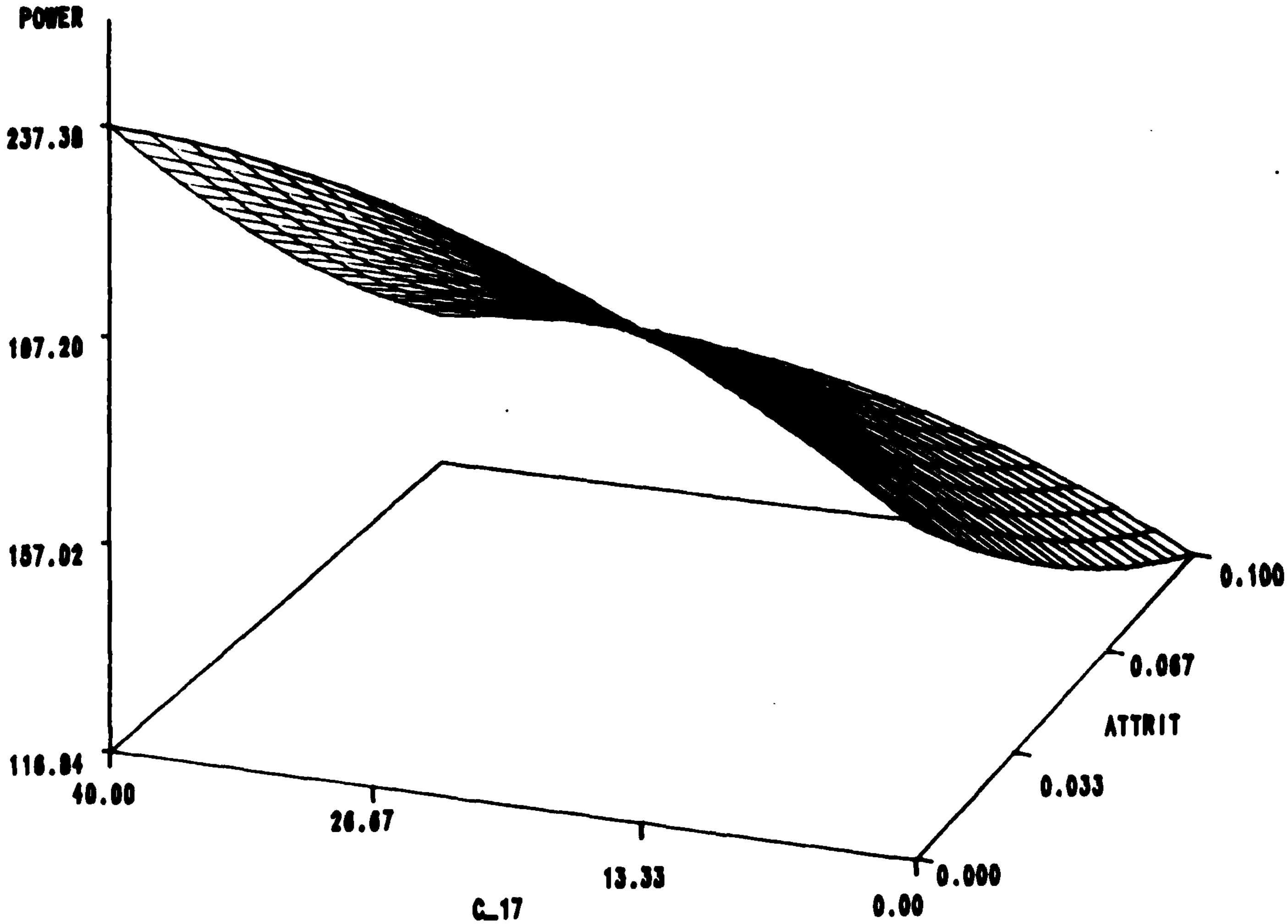


Fig. 10. Combat Power Delivered

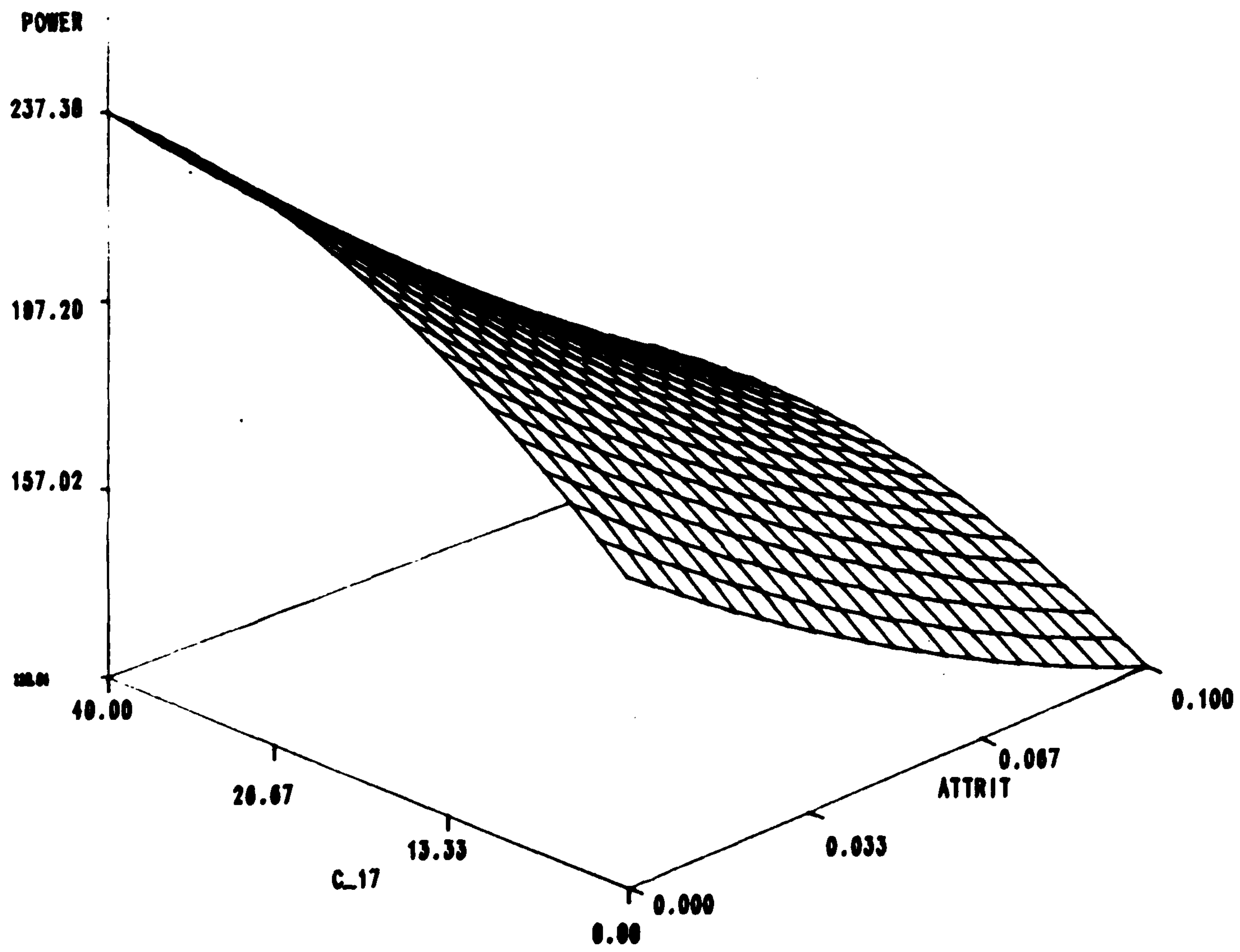


Fig. 11. Combat Power Delivered

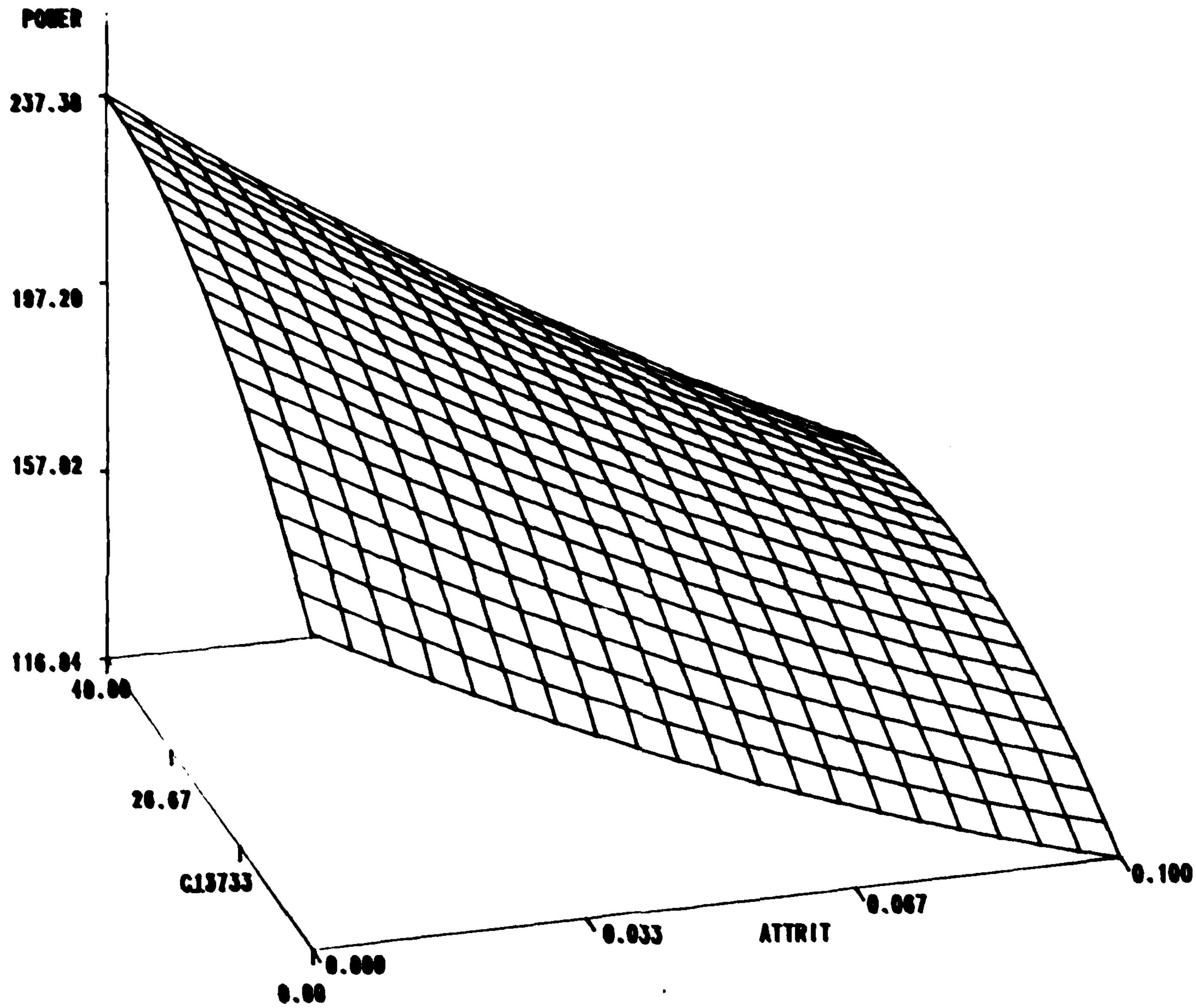


Fig. 12. Combat Power Delivered

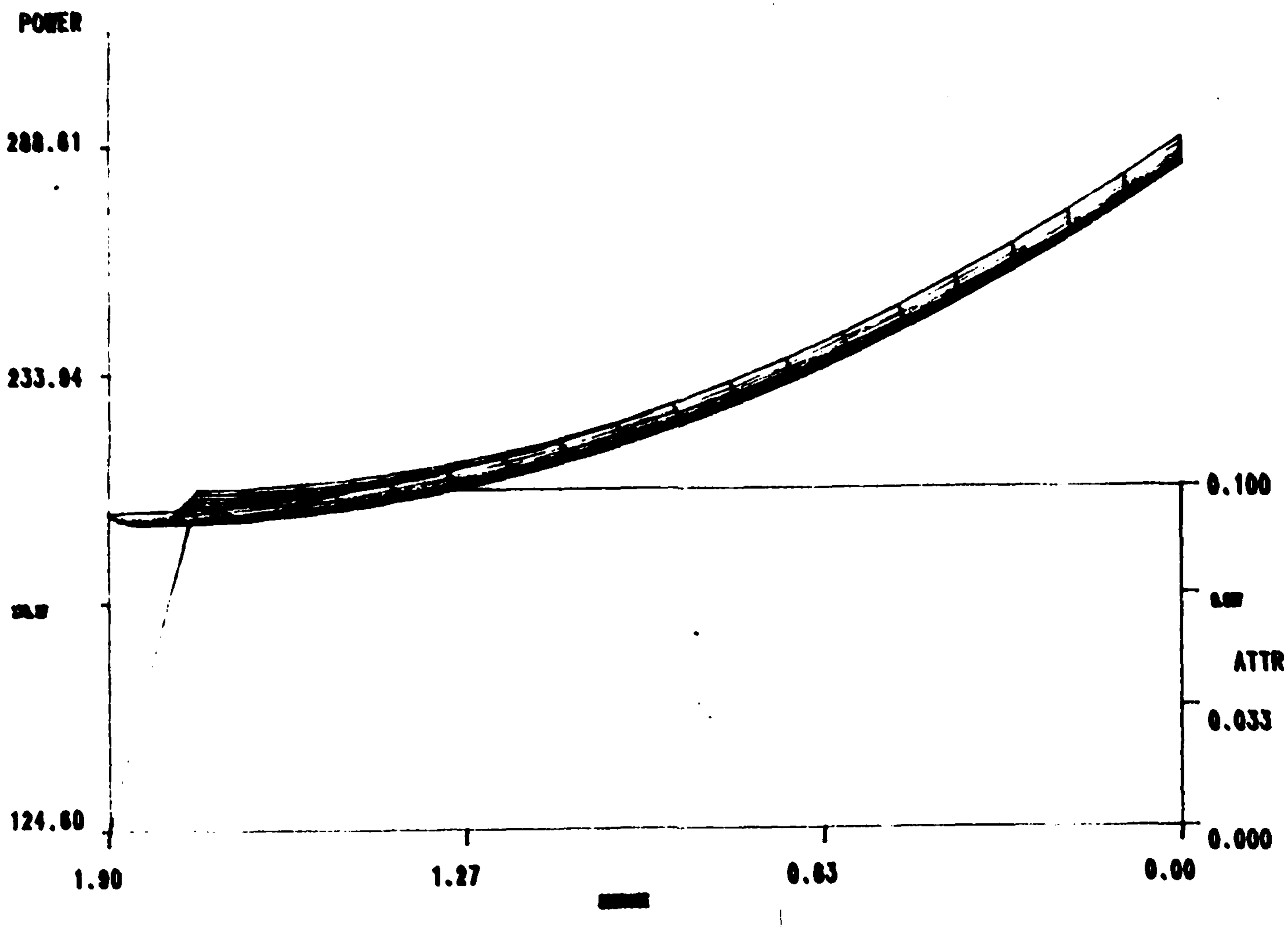


Fig. 13. Combat Power Delivered

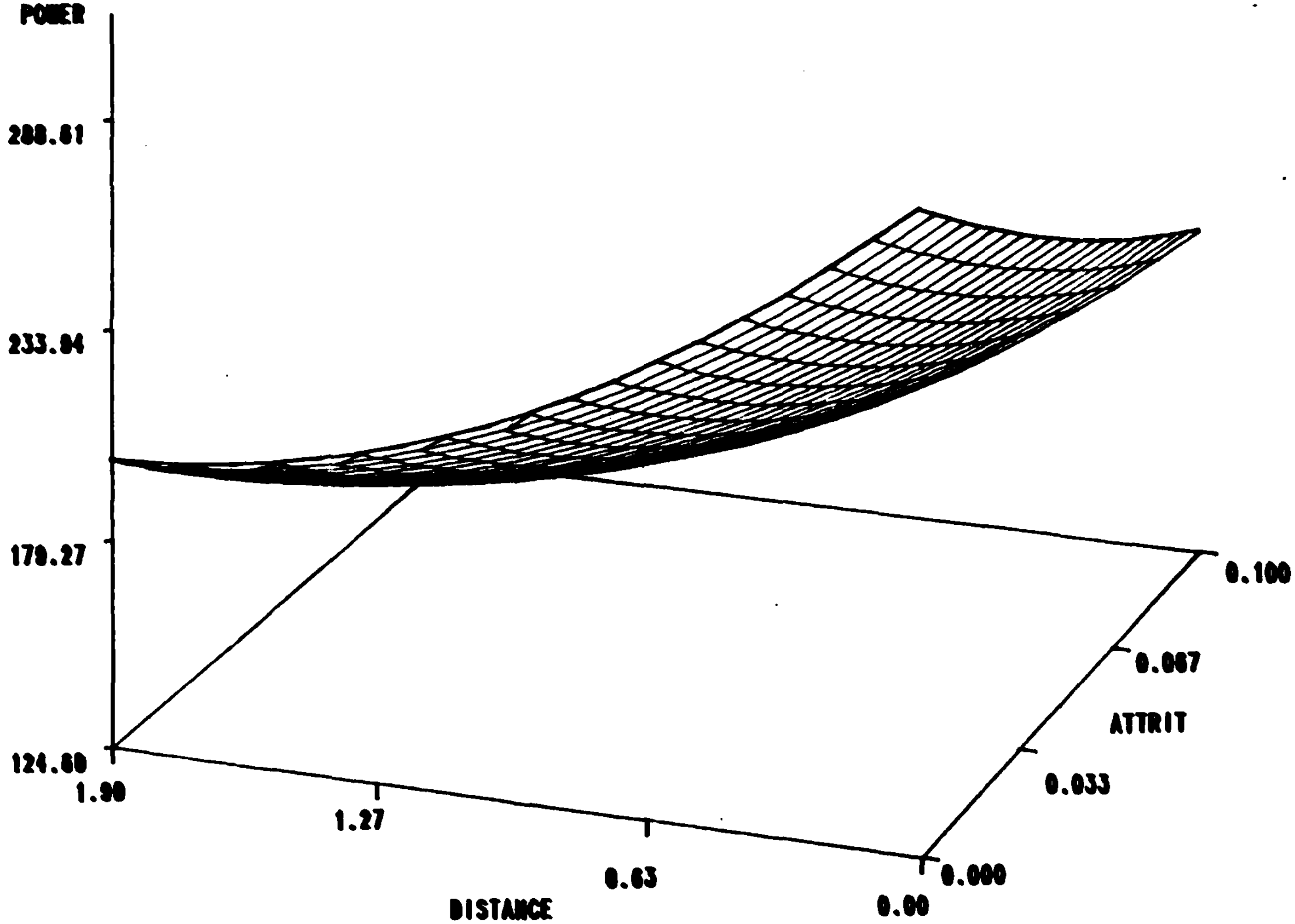


Fig. 14. Combat Power Delivered

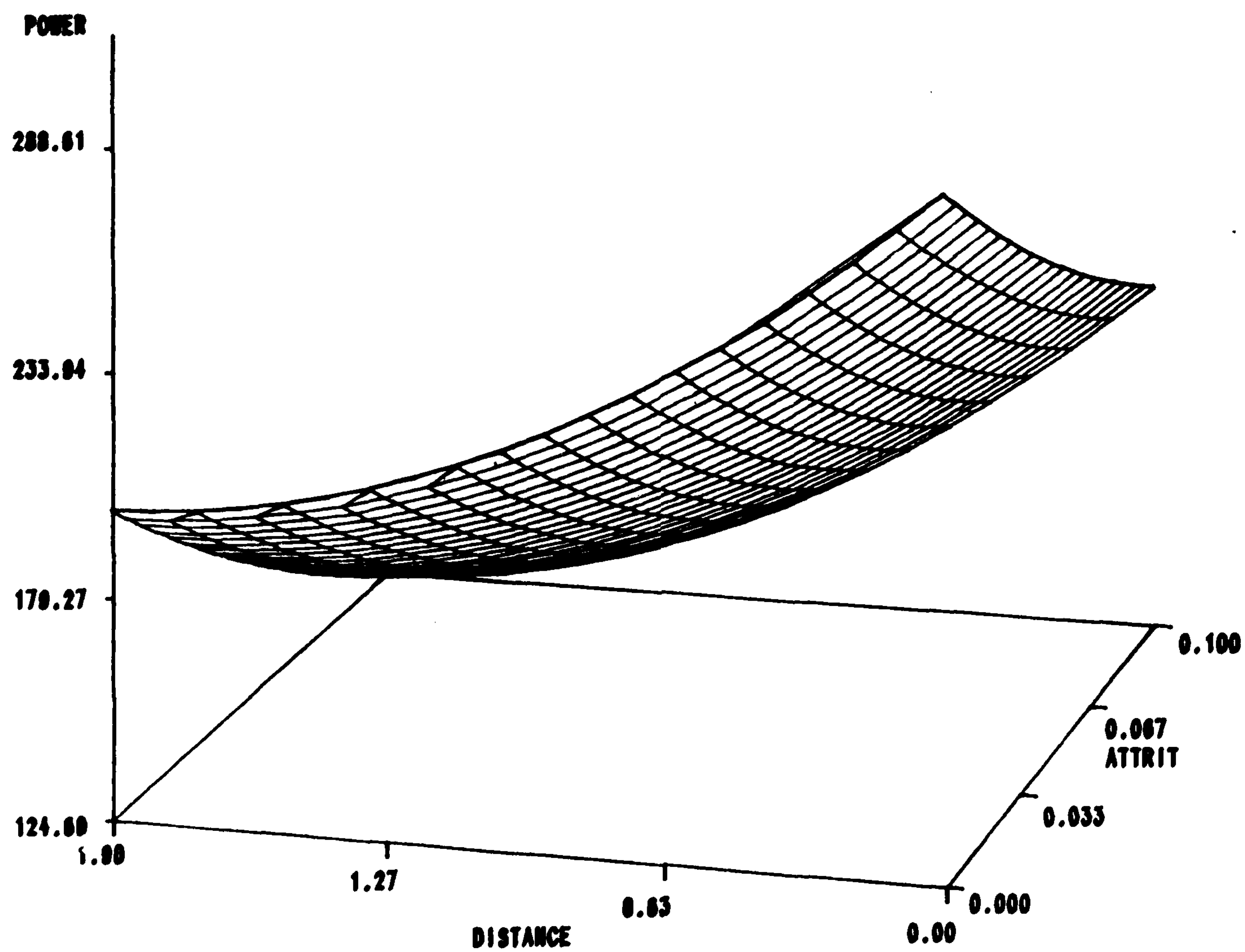


Fig. 15. **Combat Power Delivered**

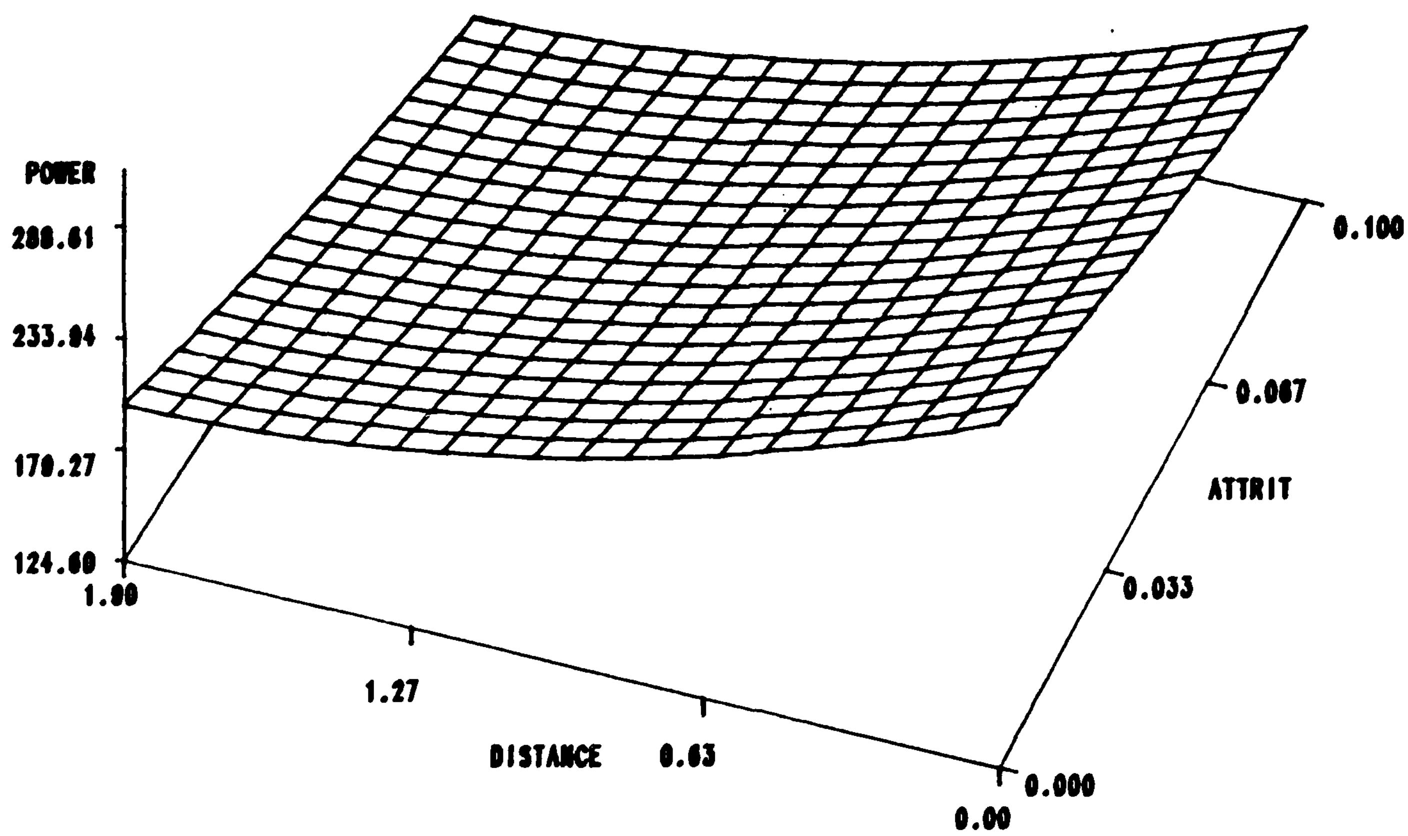


Fig. 16. **Combat Power Delivered**

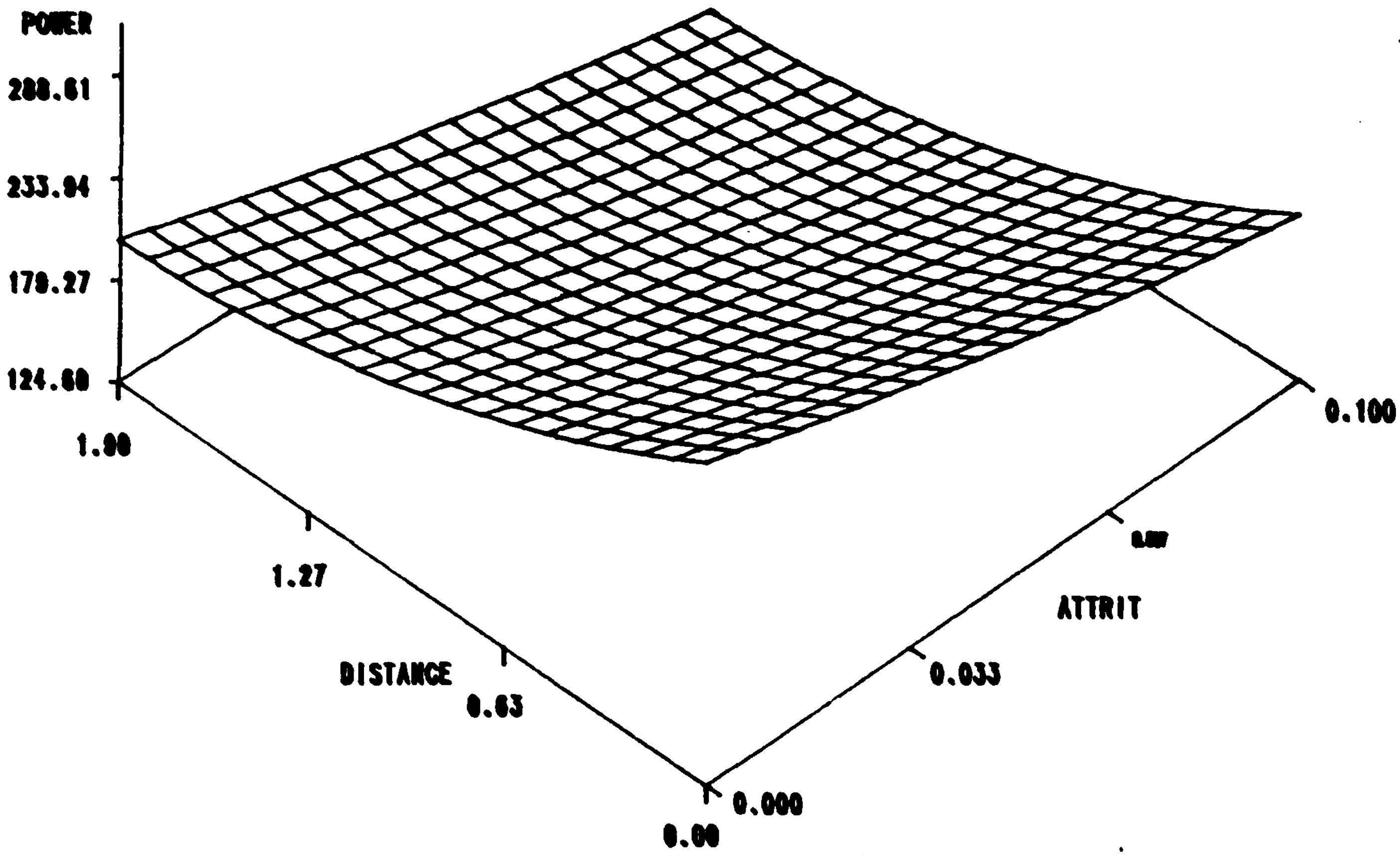


Fig. 17. Combat Power Delivered

the combat power requirements. As attrition and distance increased, the need for the C-17 became more apparent until finally approximately 35 C-17's were required to meet the established combat power value. These plots again highlight the importance of including the C-17 in an optimum airlift force structure. As the negative factors of aircraft attrition and APOD to FOL distance become more paramount, so does the requirement to include C-17's into the force structure.

These three sets of three-dimensional plots not only allow the visualization of interrelationships established with the other previously discussed methods of analysis, but they also provide a means of visualizing the tradeoffs involved in determining a proper force structure for a given scenario.

The response surface equations used for the analysis are listed in Appendix D. The first equation is the coded response surface equation. Any inputs to this equation would have to be coded using the "+1", "-1", and "0" format. Referencing the ranges applied to the variables (see Table XIII), a specific input would first have to be coded. As an example, 30 C-5 aircraft would be coded as (30-40)/20 = -0.5, with 40 equal to the center value and 20 equal to the difference between variables for the design. This coding method explains why 60 C-5's was coded as a +1 (60-40)/20 = +1. The second equation is the decoded version of the first equation so actual values,

within the applicable ranges, can be inserted into the equation without the requirement to code the input first. The third equation is the coded response equation which includes the C-5 main effect for comparison purposes.

Response Surface Limitations

There are several limitations to the effective use of these response surface equations, and they include the requirement to stay within the bounds of the variables used to generate the response equation itself. Extrapolation outside the ranges of the factor levels can provide erroneous outputs. This first limitation highlights a second restriction. If one of the factor levels does change, a new response surface must be generated. A third limitation is the potential for requiring a large number of model runs to build the response surface if a large number of factors are to be examined (29:62-63). A fourth limitation is the hazard of bringing in a relatively minor factor into the model and using that factor's minor effect as a measurement of the output of the entire model. Figure eight demonstrates this hazard by using the minor main effect of C-5 aircraft in the model to measure combat power delivered as distance between APOD and FOL increased. The third plot appears to show a decrease in the objective function value due to the combined effects of distance and C-5's. Yet, a linear program would not show a decrease in objective function value as the right hand side of a less than or equal to constraint is increased. When viewed in

the context of a maximization problem using linear programming, as the amount of a resource is increased, the objective function will either increase or remain the same with a corresponding increase in the slack value associated with that specific resource. The decreasing objective value was probably due to a combination of a less than perfectly fitting response surface and the technique of bringing in a relatively minor factor and using its output as representative output for the larger model itself.

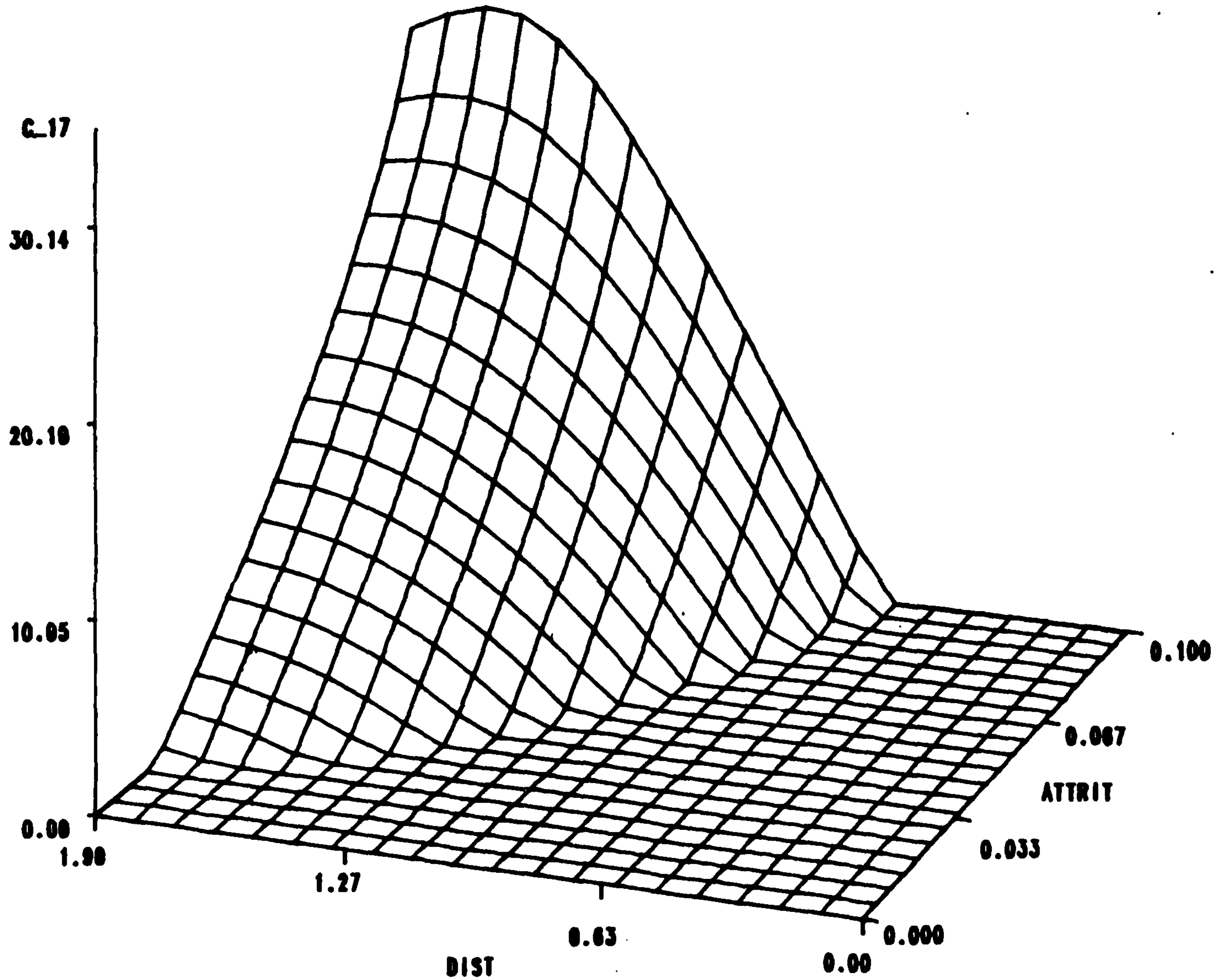


Fig. 18. **Effect of Attrition and Distance**

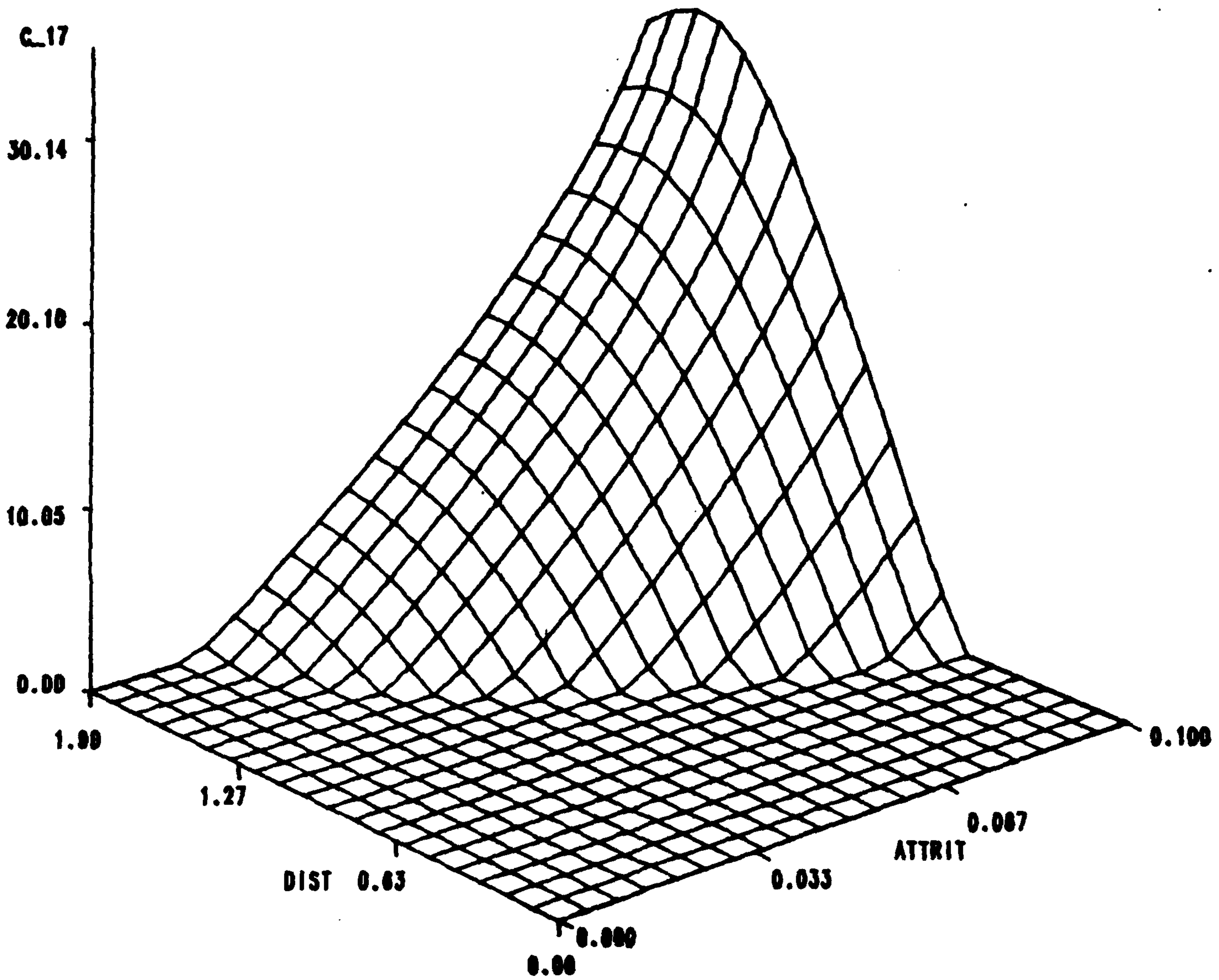


Fig. 19. **Effect of Attrition and Distance**

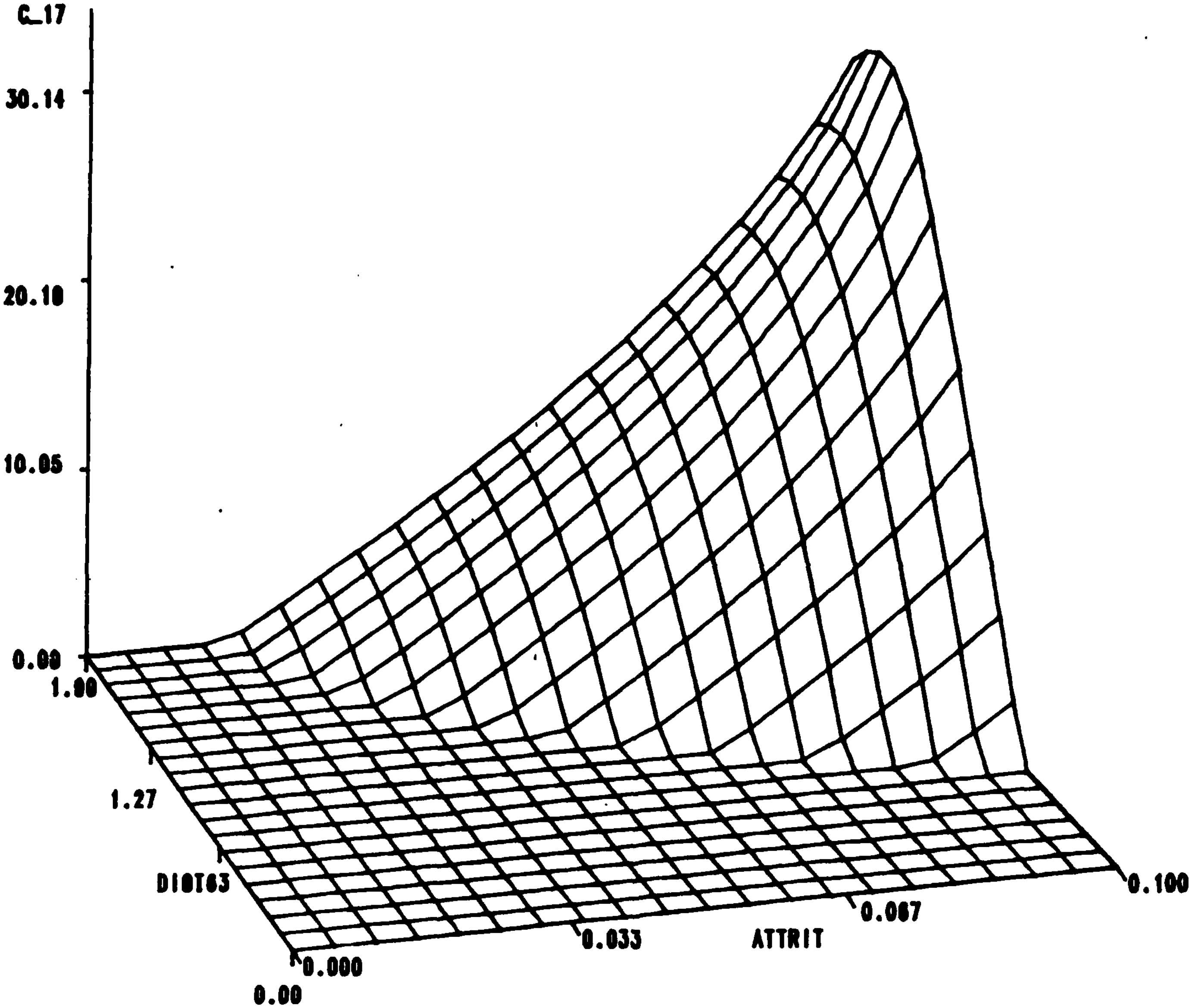


Fig. 20. **Effect of Attrition and Distance**

VI. Summary, Conclusions, and Recommendations

Summary

This thesis has further refined a methodology which combines response surface methodology and experimental design concepts to describe the output of a deterministic model. In an effort to maximize the combat power delivered to a theater commander, the interactive effects of time, MHE capability, airfield ramp space, distance between APOD and FOL, availability and capabilities of different aircraft, and the impact of aircraft attrition were examined in the model. Utilizing an appropriate experimental design, a response surface with a greater than 96 percent level of accuracy was generated for analysis purposes. The analysis demonstrated the numerous advantages and also the limitations of using response surface methodology to generate a multidimensional analysis of a given model. Even though an optimum airlift force mix may be scenario dependent, this research provides additional insight into the complex interactive factors which affect the ability of an airlift force to meet national objectives.

Conclusions

Within the ranges specified for this model, the addition of more than 20 C-5B aircraft was ineffective at increasing the combat power delivered to the objective

area. This result does not imply that the C-5B is not an important asset, but other variables such as airfield ramp capacity and MHE saturation prevented additional C-5B aircraft from becoming an important factor. For this particular scenario, with the APOD MHE limitations and ramp saturation, additional C-5 aircraft did not increase the overall effectiveness of the airlift force.

While the location of the battle may dictate which airfield(s) are designated as the FOL and APOD, the distance between these two bases was proven to be a major factor in determining the quantity of combat power delivered to the front, especially if the C-17 was not included in the airlift force structure. Minimizing the distance between the two fields can significantly affect the timely delivery of combat power, especially if units must proceed overland from the APOD to the FOL.

APOD materials handling equipment capability was also shown to be a dominant factor in effecting the delivery of combat power. The constraint for APOD MHE was consistently binding in the linear program, and the response surface equation shows APOD MHE capability as a primary factor in determining the shape of the response surface. Time permitting, the prepositioning of MHE capability in the same theater as an expected area of conflict could prove to be a major factor in expediting the flow of men and equipment to the objective area.

The C-141B was shown to be a valuable asset throughout the range of the model. C-141B usage rates and sorties generated demonstrated the high utility of the aircraft, yet the increasing levels of attrition also demonstrated the excessive vulnerability of the aircraft in a threat environment. The combination of using a vulnerable aircraft to fly on the mission most susceptible to the given threat (airdrop mission) produces an extremely low level of survivability. If the C-141B is to be utilized in a threat environment, enhancements to decrease its vulnerability, especially in the area of fuel vulnerability, must be addressed and resolved.

Aircraft attrition was demonstrated to be a major factor in determining the quantity of combat power delivered to the objective area. Combat power delivered over the 20 day operation averaged more than a 30 percent decline when the attrition factor increased to the ten percent level. Even a five percent level of attrition produced decreases in combat power of over 20 percent. The response surface verified the critical impact aircraft attrition can have upon the timely delivery of combat power. While the Air Force may not be able to dictate the susceptibility of the threat environment, it can increase the aircrafts' probability of survival by incorporating measures to reduce aircraft vulnerability to the threat.

The C-17 was shown to be a significant factor throughout the range of the scenario. Because of the aircraft's flexibility in terms of delivery capability, inclusion of the C-17 into the airlift force structure provided an added dimension to the capabilities of the total force. While the addition of 40 C-5B aircraft did not appreciably affect the quantity of combat power delivered to the objective area (Figure 8), a similar addition of 40 C-17 aircraft provided more than a 28 percent increase in the delivery of combat power (Figure 6.) The aircraft's ability to perform the airdrop mission, combined with it's capability to deliver outsized cargo directly to the objective area, were major enhancements to the current airlift force. As distance between the APOD and FOL increased, the marginal utility of each C-17 increased. APOD MHE saturation was also circumvented by the direct delivery concept associated with the C-17. The addition of this aircraft to the airlift inventory would provide added flexibility to the operational planners' capability to meet the unknown challenges of future contingency operations. The impact of unknown factors, such as the APOD MHE capability and the APOD to FOL distance, could be minimized by incorporating the inherent flexibility of the C-17.

Recommendations

Although attrition was included in the model, the impact of aircraft attrition upon the timely delivery of combat power should be examined in further detail. Rather than using an estimation of the combat power per aircraft type, a more precise formulation should be developed which examines combat power by aircraft type, mission type, and the period of deployment. Specific threat scenarios should also be included for analysis.

The variety of units to be deployed could be expanded to include the Army's High Technology Test Bed (HTTB) or a squadron of F-15 fighter aircraft.

As stated previously, an appropriate airlift force mix is scenario dependent. With the current political environment, consideration should be given to developing a scenario centered in South America. The generation of a new scenario could be expedited by the development of a matrix generator to build the linear model.

The impact of the C-130 appears to have been decreased by the high percentage of oversized and outsized cargo in this particular scenario, but further analysis is required to verify that conclusion.

While the current analysis highlighted the critical impact of the distance between the APOD and FOL, the impact of the distance from the Aerial Port of Embarkation (APOE) and the APOD could also be an important factor and may be a source for further analysis.

Appendix A

EXPERIMENTAL DESIGN *

Coded Design							"y" Value
0	0	0	1	1	1	0	144.126
0	0	0	1	1	-1	0	141.729
0	0	0	1	-1	1	0	216.846
0	0	0	1	-1	-1	0	215.086
0	0	0	-1	1	1	0	223.841
0	0	0	-1	1	-1	0	222.775
0	0	0	-1	-1	1	0	297.631
0	0	0	-1	-1	-1	0	295.355
1	0	0	0	0	1	1	194.330
1	0	0	0	0	1	-1	194.355
1	0	0	0	0	-1	1	191.664
1	0	0	0	0	-1	-1	191.664
-1	0	0	0	0	-1	-1	188.074
-1	0	0	0	0	-1	1	188.074
-1	0	0	0	0	1	1	191.356
-1	0	0	0	0	1	-1	191.356
0	1	0	0	1	0	1	185.119
0	1	0	0	1	0	-1	185.119
0	1	0	0	-1	0	-1	268.329
0	1	0	0	-1	0	1	268.329
0	-1	0	0	-1	0	1	245.704
0	-1	0	0	-1	0	-1	245.704
0	-1	0	0	1	0	-1	113.050
0	-1	0	0	1	0	1	113.050
1	1	0	1	0	0	0	164.758
1	1	0	-1	0	0	0	246.312
1	-1	0	-1	0	0	0	188.490
1	-1	0	1	0	0	0	134.811
-1	-1	0	1	0	0	0	125.035
-1	-1	0	-1	0	0	0	184.351
-1	1	0	-1	0	0	0	243.943
-1	1	0	1	0	0	0	158.782
0	0	1	1	0	0	1	166.391
0	0	1	1	0	0	-1	166.391
0	0	1	-1	0	0	-1	251.561
0	0	1	-1	0	0	1	251.561
0	0	-1	-1	0	0	1	216.923
0	0	-1	-1	0	0	-1	216.923
0	0	-1	1	0	0	-1	146.596
0	0	-1	1	0	0	1	146.596
1	0	1	0	1	0	0	154.761
1	0	1	0	-1	0	0	278.712
1	0	-1	0	-1	0	0	231.891
1	0	-1	0	1	0	0	166.817
-1	0	-1	0	1	0	0	166.149
-1	0	-1	0	-1	0	0	231.123
-1	0	1	0	-1	0	0	264.029
-1	0	1	0	1	0	0	187.584
0	1	1	0	0	1	0	215.380
0	1	1	0	0	-1	0	210.459
0	1	-1	0	0	-1	0	181.930
0	1	-1	0	0	1	0	180.004
0	-1	-1	0	0	1	0	145.173
0	-1	-1	0	0	-1	0	144.424
0	-1	1	0	0	-1	0	164.098
0	-1	1	0	0	1	0	170.559
0	0	0	0	0	0	0	194.185

* Box and Behnken (3) design

Appendix B

EXPERIMENTAL DESIGN

Non-Coded

Run #	C-5	C-17	APOD MHE	Attri-tion	Dis-tance	APOD Ramp	C-130
1	40	20	1000	.10	2	1.25	60
2	40	20	1000	.10	2	0.75	60
3	40	20	1000	.10	0	1.25	60
4	40	20	1000	.10	0	0.75	60
5	40	20	1000	0	2	1.25	60
6	40	20	1000	0	2	0.75	60
7	40	20	1000	0	0	1.25	60
8	40	20	1000	0	0	0.75	60
9	60	20	1000	.05	1	1.25	80
10	60	20	1000	.05	1	1.25	40
11	60	20	1000	.05	1	0.75	80
12	60	20	1000	.05	1	0.75	40
13	20	20	1000	.05	1	0.75	40
14	20	20	1000	.05	1	0.75	80
15	20	20	1000	.05	1	1.25	80
16	20	20	1000	.05	1	1.25	40
17	40	40	1000	.05	2	1.00	80
18	40	40	1000	.05	2	1.00	40
19	40	40	1000	.05	0	1.00	40
20	40	40	1000	.05	0	1.00	80
21	40	0	1000	.05	0	1.00	80
22	40	0	1000	.05	0	1.00	40
23	40	0	1000	.05	2	1.00	40
24	40	0	1000	.05	2	1.00	80
25	60	40	1000	.10	1	1.00	60
26	60	40	1000	0	1	1.00	60
27	60	0	1000	0	1	1.00	60
28	60	0	1000	.10	1	1.00	60
29	20	0	1000	.10	1	1.00	60
30	20	0	1000	0	1	1.00	60
31	20	40	1000	0	1	1.00	60
32	20	40	1000	.10	1	1.00	60
33	40	20	1300	.10	1	1.00	80
34	40	20	1300	.10	1	1.00	40
35	40	20	1300	0	1	1.00	40
36	40	20	1300	0	1	1.00	80
37	40	20	700	0	1	1.00	80
38	40	20	700	0	1	1.00	40
39	40	20	700	.10	1	1.00	40
40	40	20	700	.10	1	1.00	80
41	60	20	1300	.10	2	1.00	60
42	60	20	1300	.05	0	1.00	60
43	60	20	700	.05	0	1.00	60
44	60	20	700	.05	2	1.00	60
45	20	20	700	.05	2	1.00	60
46	20	20	700	.05	0	1.00	60
47	20	20	1300	.05	0	1.00	60
48	20	20	1300	.05	2	1.00	60
49	40	40	1300	.05	1	1.25	80
50	40	40	1300	.05	1	0.75	40
51	40	40	700	.05	1	0.75	40
52	40	40	700	.05	1	1.25	80
53	40	0	700	.05	1	1.25	80
54	40	0	700	.05	1	0.75	40
55	40	0	1300	.05	1	0.75	40
56	40	0	1300	.05	1	1.25	80
57	40	20	1000	.05	1	1.00	60

Appendix C

SAS REGRESSION PROGRAM

```
options linesize = 78;
filename newdata 'design.dat';
data first;
   infile newdata;
   label x1='C-5 AIRCRAFT';
   label x2='C-17 AIRCRAFT';
   label x3='APOD MHE';
   label x4='AIRCRAFT ATTRITION';
   label x5='DIST TO FOL';
   label x6='APOD RAMP SIZE';
   label x7='C-130 AIRCRAFT';
   label y='COMBAT FIRE POWER';
   input x1 x2 x3 x4 x5 x6 x7 y ;
   x11 = x1*x1;
   x12 = x1*x2;
   x13 = x1*x3;
   x14 = x1*x4;
   x15 = x1*x5;
   x16 = x1*x6;
   x17 = x1*x7;
   x22 = x2*x2;
   x23 = x2*x3;
   x24 = x2*x4;
   x25 = x2*x5;
   x26 = x2*x6;
   x27 = x2*x7;
   x33 = x3*x3;
   x34 = x3*x4;
   x35 = x3*x5;
   x36 = x3*x6;
   x37 = x3*x7;
   x44 = x4*x4;
   x45 = x4*x5;
   x46 = x4*x6;
   x47 = x4*x7;
   x55 = x5*x5;
   x56 = x5*x6;
   x57 = x5*x7;
   x66 = x6*x6;
   x67 = x6*x7;
   x77 = x7*x7;
proc stepwise;
   model y = x1 x2 x3 x4 x5 x6 x7 x11 x12
             x13 x14 x15 x16 x17 x22 x23 x24
             x25 x26 x27 x33 x34 x35 x36 x37
             x44 x45 x46 x47 x55 x56 x57 x66 x67
             x77/maxr;
```

```
/*          Regress on ALL Variables                    */

proc reg data = first;
   model y = x1 x2 x3 x4 x5 x6 x7 x11 x12
             x13 x14 x15 x16 x17 x22 x23 x24
             x25 x26 x27 x33 x34 x35 x36 x37
             x44 x45 x46 x47 x55 x56 x57 x66 x67 x77;
   output out=resids p=pred r=resid;
      data residul;
        set resids;
        error1 = resid/pred;
        err1 = abs(error1);
   proc plot data = residul;
        plot resid*pred = "R"/vref=0;
   proc plot data = residul;
        plot  error1*pred= "E"/vref= 0;
   proc print;
        var error1 y pred;
   proc means mean min max;
        var y pred err1;

/*             Regress on Selected Model                   */
proc reg data = first;
   model y =  x2 x3 x4 x5 x15 x22 x24 x25
              x34 x35 x44 x55/corrb;
   output out = resoids2 p = pre r = res;
      data residual;
        set resoids2;
        error = res/pre;
        err = abs(error);
   proc plot data = residual;
          plot error*pre/vref=0;
   proc print;
        var error x2 x3 x4 x5 x15 x22 x24
                  x25 x34 x35 x44 x55 y pre;
   proc means mean min max ;
        var  x2 x3 x4 x5 x15 x22 x24 x25
             x34 x35 x44 x55 y pre err;
```

Appendix D

RESPONSE SURFACE EQUATIONS

Coded Response Surface Equation (12 Variables):

$$y = 189.986739 + 22.250625X_2 + 12.789041X_3 - 38.021625X_4 - 43.942458X_5 - 5.95075X_1X_5 - 12.717293X_2^2 - 6.715X_2X_4 + 12.361X_2X_5 - 3.71075X_3X_4 - 8.7935X_3X_5 + 5.026456X_4^4 + 23.529331X_5^5$$

Converted Response Surface Equation (12 Variables):

$$y = 174.97222 + 2.101961X_2 + .084311X_3 - 579.80743X_4 - 62.148955X_5 - .297538X_1X_5 - .031793X_2^2 - 6.715X_2X_4 + .61805X_2X_5 - .247383X_3X_4 - .02931167X_3X_5 + 2010.582608X_4^2 + 23.529332X_5^2$$

Coded Response Surface Equation (To include C-5 main effect, 19 Variables):

$$y = 194.473154 + .779541X_1 + 22.250625X_2 + 12.789041X_3 - 38.021625X_4 - 43.942458X_5 + 1.234375X_6 - 3.150048X_1^1 - 2.447X_1X_3 - 5.95075X_1X_5 - 14.119298X_2^2 + 2.355625X_2X_3 - 6.715X_2X_4 + 12.361X_2X_5 - 3.299173X_3^3 - 3.71075X_3X_4 - 8.7935X_3X_5 + 1.569875X_3X_6 + 3.624451X_4^4 + 22.127326X_5^5$$

Bibliography

1. Acuff, Maj. Steven D. and Maj. Jeffery D. Wise. Introduction of the C-17 Into the Military Airlift Command Airlift Force. MS Thesis, Air Command and Staff College, Air University (ATC), Maxwell AFB, AL 36112.

2. Andrean, Col Charles M. Army C-17 Test Manager. Personal Interview, Aeronautical Systems Division/XRS. Wright-Patterson AFB, OH 45433. 5 Sep 1986.

3. Box, G.E.P. and D.W. Behnkin." Some New Three Level Designs for the Study of Quantitative Variables", Technometrics, 2: 455-474 (November 1960).

4. Bullard, Lt Col Edward M. The Airlift Delemma: An Update. Air War College, Washington D.C., May 1985 (AD-A159 307).

5. Carlton, General Paul K. "Strategic Airlift: A Cargo Capability Shortfall", Air University Review: 2-10 (November-December 1975).

6. Conahan, Frank C. "Performance Capabilities of the C-5 and C-17 Cargo Aircraft", July 1984. General Accounting Office. Washington D.C. National Security and International Affairs Division. (AD-A159 307)

7. Cook, Capt James C. Optimizing Force Deployment and Force Structure for the Rapid Deployment Force. MS Thesis, School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB, OH, March 1984.

8. Department of the Air Force. U.S. Air Force Airlift Master Plan, Scott AFB, Il, Headquarters Military Airlift Command, 29 September 1983.

9. Department of Defense. Sealift Study. Table D.12 Washington: Government Printing Office.

10. Dunnigan, James F. How to Make War. A Comprehensive Guide to Modern Warfare. New York: Quill, 1983.

11. Graney, Capt Robert E. An Optimization Methodology For Multi-Criteria Comparison Using Response Surface Methods and Mathematical Programming. MS Thesis, School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB, OH, Dec 1984.

12. Hill, Maj James. HQ MAC/XOSS. Telephone Conversation 30 July 1986.

13. Hooten, Col Byron R. The C-17: We Need It Yesterday. U.S. Army War College, Carlisle Barracks, Pennsylvania, April 1985.

14. Ishihara, Capt Brian K. Evaluating Experimental Designs for Fitting Response Surfaces of Deterministic Models, MS Thesis, School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB, OH, December 1985.

15. Manacapilli, Capt Thomas W. A Methodology for Identifying Cost Effective Strategic Force Mixes. MS Thesis, School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB, OH, Dec 1984.

16. Maroney, Maj Michael. Aeronautical Systems Division/XRS, personal interview, Wright-Patterson AFB, OH 6 Nov 1986.

17. May, Lt Col Gary B. The Impact of Materials Handling Equipment On Airlift Capabilities. MS Thesis, Air Command and Staff College, Air University, Maxwell AFB, Alabama, 1983.

18. Meitzler, Thomas D. An Application of the Response Surface Methodology to a Readiness-Analysis Problem, MS Thesis, School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB, OH, Dec 1984.

19. Military Standard-2089. Aircraft Nonnuclear Survivability Terms. Washington: Department of Defense, 21 July 1981.

20. Montgomery, Douglas C. Design and Analysis of Experiments. New York. John Wiley and Sons, 1976.

21. Moseman II, Lloyd K. "The Air Force's Solution to the Airlift Shortfall", Defense Management Journal, Vol. 19, No. 2: 14 (Second Quarter 1983).

22. Myers, Raymond H. Response Surface Methodology, Boston Massachusetts: Allyn and Bacon, Inc., 1971.

23. Neter, John and William Wasserman, Applied Linear Statistical Models, Richard D. Irwin, Inc., Homewood, Illinois, 1974.

24. Pilsch, Col Thomas D. "The Airlift Master Plan: Evolution and Implementation". Defense Management Journal, Vol. 20, No. 4: 28-29 (Fourth Quarter 1984).

25. Redman, John, Aeronautical Systems Division/XRS, personal interview, Wright-Patterson AFB, OH, 6 Nov 86.

26. SAS Institute Inc. SAS User's Guide: Statistics, Version 5 Edition, Cary, NC: SAS Institute Inc., 1985, 956 pp.

27. SAS Institute Inc. SAS/Graph User's Guide: Statistics, Version 5 Edition, Cary, NC: SAS Institute Inc., 1985, 596 pp.

28. SAS Institute Inc. SAS/OR User's Guide, Version 5 Edition, Cary, NC: SAS Institute Inc., 1985, 347 pp.

29. Smith, Lt Col Palmer W. and Joseph M. Mellichamp. "A Methodology for Multidimensional Impact Analysis for Military Problems," Proceedings of the 53rd MORS. June 1985.

30. Soligan, Major James N. Direct Delivery. MS Thesis, Air Command and Staff College, Air University (ATC), Maxwell AFB, AL 36112 (AD-B092 819).

31. Sparrow, 1Lt Kalla J. An Interactive Computer Package for Use with Simulation Models which Performs Multi-dimensional Sensitivity Analysis by Employing the Techniques of Response Surface Methodology, MS Thesis, School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB, OH, Dec 1984.

32. Stone, Maj John W. The C-17: An Attempt at Increased Airlift Versatility, U.S. Army Command and General Staff College, Fort Leavenworth, Kansas, Dec 1985.

33. Tate, Capt David O. DEPLOY: An Interactive Goal Programming Model for the Rapid Deployment of Armed Forces. MS Thesis, School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB, OH, December 1984.

34. U.S. Congress. Air Force Report: Report to the 96th Congress, Fiscal Year 1981, Kelly AFB, Texas: Air Force Service Information and News Center: 17 (February 1980).

Vita

Major Raymond F. Haile was born on March 10, 1952. He attended the U.S. Air Force Academy and graduated in 1974 with a B.S. degree in Management, and later completed an M.A. degree in Management. After completion of Undergraduate Pilot Training at Columbus AFB, he was assigned to McGuire AFB, New Jersey. While stationed at McGuire AFB he held various positions including Assistant Chief of Squadron Standardization and C-141 Lead Airdrop Flight Examiner. He was subsequently assigned to the 443rd MAW at Altus AFB, Oklahoma, where he maintained flight examiner, simulator instructor, and lead airdrop status while assigned to the 57th MAS, 443rd Technical Training Squadron, and 443rd MAW. He was also a senior command post controller, Chief of the 443rd Wing Mission Briefing Team and the 1980 Air Force Recipient of the Aviator's Valor Award.

Permanent Address: 414 Spruce Street
Baird, Texas 79504

END
11-87
DTIC